INDIVIDUAL TREE VOLUME EQUATIONS FOR ASPEN IN MICHIGAN

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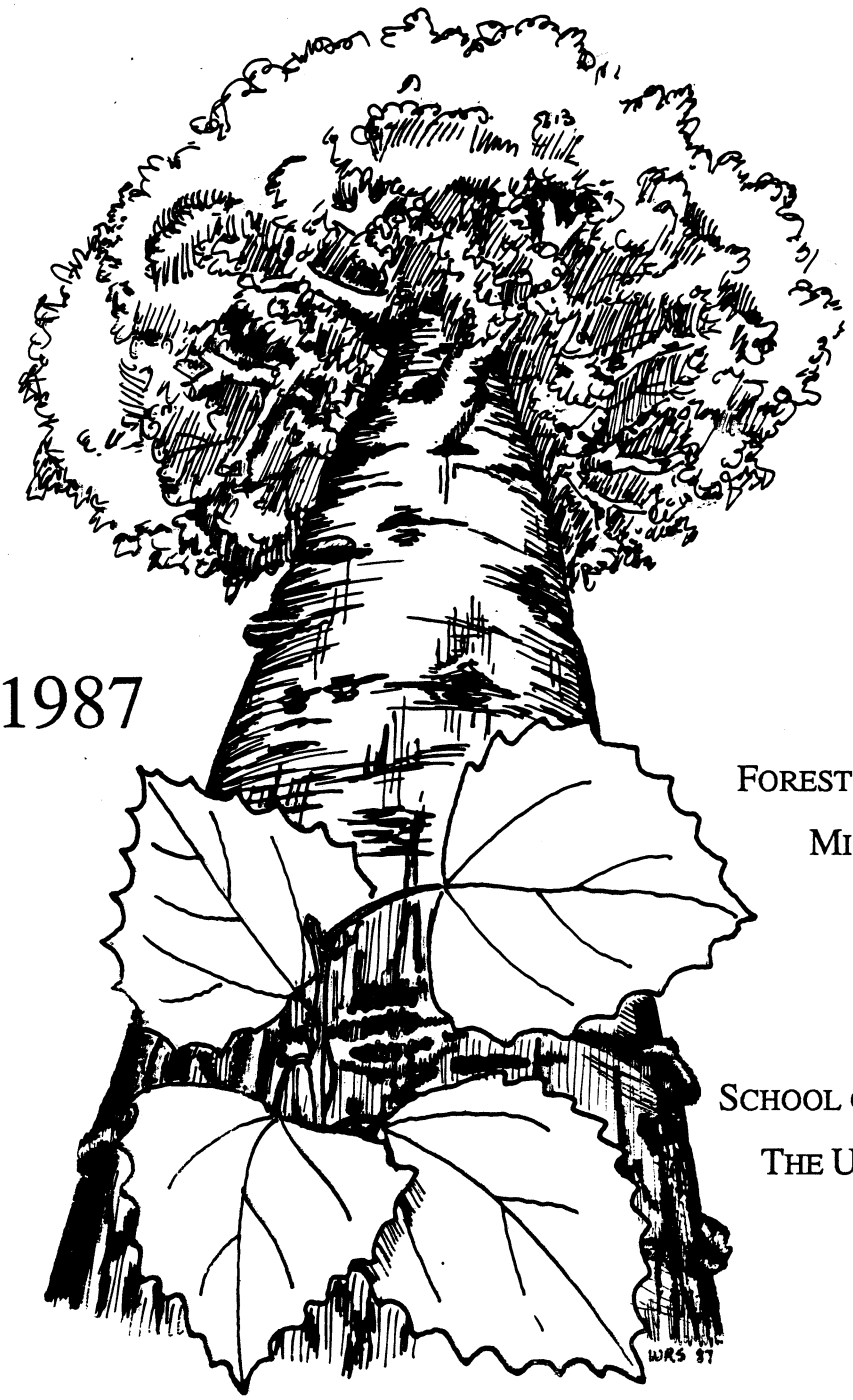

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FOREST MANAGEMENT DIVISION

MICHIGAN DEPARTMENT OF

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Forest Management Division
Michigan Department of Natural Resources
and
School of Natural Resources
The University of Michigan

Management Summary

New pulpwood, sawtimber, and residual pulpwood cubic-foot individual tree volume equations were developed for aspen in Michigan. Data used to develop these equations were collected from 24 aspen stands in Michigan (12 stands each from the Upper and Lower Peninsulas). Four stands were sampled from each of the 6 state forests in Michigan.

Examination of coefficients of determination (R^2), standard errors of the estimate ($s_{y \cdot x}$), and independent validation data sets for a series of linear and nonlinear regression equations indicated that: (1) nonlinear equations were most accurate for pulpwood and sawtimber cubic-foot volume, and (2) linear equations were most accurate for residual pulpwood cubic-foot volume. The residual pulpwood volume equations using diameter at breast height (DBH) and height independent variables yielded somewhat higher R^2's, lower $s_{y \cdot x}$'s, and more accuracy than the equations using height independent variables. However, the differences between the 2 equations is relatively small, indicating that the use of the simpler height equations is adequate for most cruising situations. The new pulpwood volume equations will usually yield rough cord volume per acre estimates from 5% lower to 8% higher and from 12-18% higher than estimates based on Table 6 in Gevorkiantz and Olsen (1955) and Table 1 in Ek and Droessler (1986), respectively, depending on stand DBH.

We recommend the use of the following individual tree volume equations in most cruising situations for aspen:

1. Pulpwood volume

$$\hat{V}_P = 0.2075+0.04384D^{1.8713}{}_{PH}{}^{0.8546}$$

2. Sawtimber volume

$$\hat{V}_S = 0.3617+0.09988D^{1.5648}{}_{SH}{}^{0.9388}$$

3. Residual pulpwood volume

$\hat{V}_{RP} = \hat{P} \cdot \hat{V}_P$, where

$$\hat{P} = -0.2752 + 0.9937 \cdot \frac{RH}{PH} + 0.1692 \cdot \frac{1}{RH} + 0.02340 \cdot \frac{PH}{SH}$$

In the above equations, PH is pulpwood merchantable height in 100-in. sticks to an approximate 3.6-in. top diameter limit, SH is sawtimber merchantable height to an approximate 7.6-in. top diameter limit, RH is the residual number of pulpwood sticks above and beyond sawtimber sticks, and $\hat{P}$ is the predicted proportion of residual pulpwood volume. For trees with both sawtimber and pulpwood volume, we recommend that sawtimber volume be determined using $\hat{V}_S = \hat{V}_P - \hat{V}_{RP}$. For trees with just sawtimber, Equation 2 above should be used. Pulpwood and residual pulpwood rough cord volumes can be determined from the respective cubic-foot volumes using appropriate cu.ft./cd. conversion values.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

MICHIGAN
DEPARTMENT OF NATURAL RESOURCES
FOREST MANAGEMENT DIVISION

SUBJECT - INDIVIDUAL TREE VOLUME EQUATIONS

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TITLE - Individual Tree Volume Equations for Aspen in Michigan.

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Background

Composite individual tree volume tables have been developed for the Lake States by Gevorkiantz and Olsen (1955). Ek and Droessler (1986) stated that Table 6 of Gevorkiantz and Olsen gives substantial overestimates of volumes of large trees and modified this table for volumes to a constant 4-in. top diameter limit. The tables of Gevorkiantz and Olsen or some modifications of them are still widely used in Michigan for aspen.

Purpose

The purpose of this paper is to present new pulpwood, sawtimber, and residual pulpwood cubic-foot individual tree volume equations and tables for aspen in Michigan. Rough cord volume equations and tables are also given for pulpwood and residual pulpwood.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 1381 trees from 24 stands as follows:

1) 570 trees from 12 stands in the Upper Peninsula (i.e., 4 stands each in the Copper, Escanaba River, and Lake Superior state forests), and
2) 591 trees from 12 stands in the Lower Peninsula (i.e., 4 stands each in the Mackinaw, Au Sable, and Marquette state forests).

Measurements were taken on 728 bigtooth aspen and 653 trembling aspen trees. Stands were selected from the six forests to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May-August, 1986.

For the 24 stands, site index varied from 51 to 79, age varied from 47 to 70 years, basal area/acre varied from 70 to 186 sq.ft., average DBH varied from 7.7 to 11.9 in., average total height varied from 52.2 to 77.5 ft., and average merchantable height to an approximate 3.6-in. minimum top diameter varied from 3.4 to 7.7 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5 ft.), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud Dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987).

Merchantable height to an approximate 3.6-in. minimum top diameter is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum inside bark top diameter no smaller than 3.6 inches. This minimum top diameter was decreased for trees where the last stick had a minimum top diameter of 3.6-in. at a length of at least 6 ft. and a full 100-in. stick could be cut from the tree. For felled trees, the last stick sometimes had a minimum top diameter less than 3.6 inches. Merchantable height to an approximate 7.6-in. minimum top diameter is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 7.6 inches.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into two pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the two volumes. Pulpwood and sawtimber were determined by summing up the volumes of sticks to approximate 3.6-in. and 7.6-in. top diameter limits, respectively. Residual pulpwood volume above and beyond sawtimber volume was determined as the difference between pulpwood and sawtimber volumes. Pulpwood volumes in rough cords were obtained by dividing cubic-foot volumes by 73-80 cu.ft./cd., depending on the average DOB of all sticks for trees with merchantable heights varying from 1-9 sticks (Taras 1956, Avery and Burkhart 1983), and multiplying the result by 0.96 to compensate for the extra 4 in. of stick length beyond 8 feet. Residual pulpwood volumes in rough cords were obtained by dividing cubic-foot volumes by 75 cu.ft./cd. and multiplying by 0.96.

Individual tree volume was regressed on various forms of tree height and DBH using multiple linear regression and nonlinear regression.

Results

The data set used to develop the regression equations consisted of approximately 80% of the trees in each of the 24 stands, yielding a total of 1161 trees. Individual tree pulpwood volume equations of the form $V=\beta_0+\beta_1 D^2H$ were developed separately for trembling (544 trees) and bigtooth (617 trees) aspen for each state forest, where V is cubic-foot volume, D is DBH, and H is pulpwood merchantable height. There were no significant differences between the equations of the 2 aspen species using the paired comparison t-test for intercepts (β_0) and regression coefficients (β_1) with a level of significance $\alpha=0.05$. Therefore, the data for both species were pooled before developing volume regression equations.

The pulpwood volume equations were based on 1161 trees with average DBH = 9.7 in. (range: 4.6 to 16.7), average merchantable pulpwood height = 5.5 sticks (range: 1 to 9), average total height = 64.8 ft. (range: 34.3 to 95.8), and average cubic-foot volume = 14.80 cu.ft. (range: 0.81 to 58.46). A pulpwood tree had to have a DBH $\geq$4.6" with at least one 100-in. stick having an approximate top diameter $\geq$3.6 inches.

The sawtimber volume equations were based on 802 trees with an average DBH = 10.8" (range: 8.0 to 16.7), average merchantable sawtimber height = 2.8 sticks (range: 1 to 7), and average cubic-foot sawtimber volume = 12.20 (range: 2.78 to 54.49). A sawtimber tree had to have at least one 100-in. stick with a minimum top diameter $\geq$7.6 inches.

The residual volume equations were based on the same 802 trees used to develop the sawtimber volume equations. The number of residual pulpwood sticks varied from 2 to 6, 2 to 6, 1 to 5, 1 to 5, 1 to 4, and 1 to 3 for 1, 2, 3, 4, 5, and 6 sawtimber sticks, respectively. Two trees with 7 sawtimber sticks had 2 residual pulpwood sticks.

Pulpwood and sawtimber volume prediction equations

Individual tree pulpwood volume equations of the form $V=\beta_o+\beta_1 D^2H$ were developed separately for each of the 24 stands. There were no significant differences between the stands in the U.P. and the stands in the L.P. using the one-way analysis of variance for intercepts (β_o) and the regression coefficients (β_1) with a level of significance $\alpha=0.05$. Therefore, all of the data from the 24 stands were pooled for each type of volume equation to develop one pooled prediction equation for Michigan.

A comparison of various multiple linear regression and nonlinear regression equations based on goodness-of-fit and simplicity indicated that the following

nonlinear prediction equation compared favorably to all other equations examined for pulpwood and sawtimber volumes:

$$\hat{V} = \hat{\beta}_o + \hat{\beta}_1 D^{\hat{\beta}_2} H^{\hat{\beta}_3}$$

where $\hat{V}$ is predicted volume, D is DBH in inches, and H is merchantable height in 100-in. sticks to an approximate 3.6-in. top diameter limit (PH) or merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit (SH) for pulpwood and sawtimber volumes, respectively. $\hat{\beta}_o$ is the sample intercept or regression constant, and $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ are the sample regression coefficients related to the independent variables.

Table 1 shows the pulpwood and sawtimber volume prediction equations along with sample sizes (n), standard errors of the estimate ($s_{y \cdot x}$), and coefficients

Table 1. Estimated parameters ($\hat{\beta}_o$, $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$), sample sizes (n), standard errors of the estimate ($s_{y \cdot x}$), and coefficients of determination (R^2) for the pulpwood and sawtimber cubic-foot volume prediction equations.

Prediction Equation	$\hat{\beta}_o$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	n	$s_{y \cdot x}$	R^2
(1) Pulpwood[a]	0.2075	0.04384	1.8713	0.8546	1161	1.13	0.981
(2) Sawtimber[b]	0.3617	0.09988	1.5648	0.9388	802	0.64	0.994

[a] $\hat{V} = 0.2075 + 0.04384 D^{1.8713} H^{0.8546}$.
[b] $\hat{V} = 0.3617 + 0.09988 D^{1.5648} H^{0.9388}$.

of determination (R^2). Pulpwood and sawtimber cubic-foot volume tables are shown in Table 2 and 3, respectively, in the Appendix. A pulpwood rough cord volume table is given in Table 4 in the Appendix. The values in this table were obtained by dividing the pulpwood volume values in Table 2 by 73, 75, 76, 77, 78, 79, and 80 cu.ft./cd. for trees with 1, 2-3, 4, 5, 6, 7, and 8-10 sticks, respectively, and multiplying the result by 0.96.

The values in Table 4 are greater and less than the values in Table 6 of Gevorkiantz and Olsen (1955) for DBH's less than 9 in. and greater than 12 in., respectively, with mixed results for DBH's between 9 and 12 inches. The values in Table 4 are greater than the values in Table 1 of Ek and Droessler (1986). In general, the values of Table 4 are between the values of Gevorkiantz and Olsen and those of Ek and Droessler for DBH's greater than 12 inches. The tables of Gevorkiantz and Olsen and Ek and Droessler used a cubic-foot per rough cord value of 79 while we used values ranging from 73 for 1 stick trees to 80 for trees with 8 or more sticks. If we used the value 79, our values in Table 4 would be smaller for smaller diameters.

Residual pulpwood volume prediction equations

Residual pulpwood volume in a sawtimber tree was estimated using two methods: 1) Use of a prediction equation for residual volume; and 2) multiplication of the estimated proportion of residual pulpwood volume in a sawtimber tree by the estimated pulpwood volume in the tree.

Method 1 -- Two prediction equations were developed for residual pulpwood cubic-foot volume:

1. Height independent variables

(3) $$\hat{V}=1.2100+1.5758 \cdot RH-0.1842 \cdot \ln SH$$
$$R^2=0.916,\ s_{y \cdot x}=0.51,\ n=802$$

2. Height and diameter independent variables

(4) $$\hat{V}=3.4630+1.6367 \cdot RH-0.5497 \cdot \ln SH-22.5483 \cdot \frac{1}{D}$$
$$R^2=0.922,\ s_{y \cdot x}=0.49,\ n=802$$

where $\hat{V}$ is predicted residual pulpwood volume and RH is the residual number of pulpwood sticks above and beyond sawtimber sticks.

Table 5 in the Appendix shows residual cubic-foot volumes for various numbers of sawtimber and residual pulpwood sticks based on Equation 3. Table 6

in the Appendix shows residual rough cord volumes for various numbers of sawtimber and residual pulpwood sticks. Volumes from Table 5 were divided by 75 cu.ft./cd. and then multiplied by 0.96 to obtain Table 6 values. A residual cubic-foot or rough cord volume table can also be developed from Equation 4 for various values of SH, RH, and D.

Method 2 -- Two prediction equations were developed for proportion of residual pulpwood volume in a sawtimber tree:

1. Height independent variables

$$\hat{P}=-0.2752+0.9937\cdot\frac{RH}{PH}+0.1692\cdot\frac{1}{RH}+0.02340\cdot\frac{PH}{SH} \quad (5)$$

$R^2=0.988$, $s_{y\cdot x}=0.0212$, n=802

2. Height and diameter independent variables

$$\hat{P}=-0.3682+0.8055\cdot\frac{RH}{PH}+0.1208\cdot\frac{1}{RH}+0.02729\cdot\frac{PH}{SH}+2.1513\cdot\frac{1}{D} \quad (6)$$

$R^2=0.992$, $s_{y\cdot x}=0.0174$, n=802

where $\hat{P}$ is the predicted proportion of residual pulpwood volume and SH is the number of sawtimber sticks. Table 7 in the Appendix gives values of $\hat{P}$ from Equation 5 for various numbers of sawtimber and residual pulpwood sticks.

Predicted residual pulpwood volume can be obtained by multiplying either Equation 5 or 6 times Equation 1. A residual pulpwood cubic-foot volume table can be developed for various values of SH, RH, and D. A separate table could be developed for each value of D.

For sawtimber trees with residual pulpwood volume, sawtimber cubic-foot volume can be obtained by subtracting predicted residual pulpwood volume (using Methods 1 or 2) from predicted pulpwood volume (Equation 1).

Validation

The data set used to validate the prediction equations consisted of the other approximately 20% of the trees in each of the 24 stands, yielding a total of 220 pulpwood trees and 144 sawtimber trees. For each volume equation, the

average relative error as a percent ($\overline{RE}$) was calculated for the pooled data set where

$$\overline{RE} = \sum_{i=1}^{n} RE_i \Big/ n$$

and $RE_i = [(\hat{V}_i - V_i)/V_i]x100$, $\hat{V}_i$ and V_i are the predicted and actual volumes for the i^{th} tree, and n is the number of trees in the pooled data set. The relative error as a percent for the sum of the predicted volumes was also calculated where

$$RE_S = \left[\left(\sum_{i=1}^{n} \hat{V}_i - \sum_{i=1}^{n} V_i\right) \Big/ \sum_{i=1}^{n} V_i\right] x100$$

and $\sum_{i=1}^{n} \hat{V}_i$ and $\sum_{i=1}^{n} V_i$ are the sum of the predicted and actual volumes, respectively.

For 220 trees, $\overline{RE}$ was 0.30% (range: -21.9 to 23.5%) and RE_S=-0.07% for the pulpwood cubic-foot volume equation (Equation 1).

For 144 trees, $\overline{RE}$ was -0.23% (range: -14.7 to 13.1%) and RE_S was -0.07% for the sawtimber cubic-foot volume equation (Equation 2).

For 144 trees, $\overline{RE}$ was -0.34% (range: -19.4 to 29.0%) and -0.30% (range: -17.6 to 34.8%), and RE_S was -1.20 and -1.10%, for the residual pulpwood cubic-foot volume equations based on height (Equation 3) and height and diameter (Equation 4) independent variables, respectively (Method 1). $\overline{RE}$ was -0.79% (range: -32.2 to 51.94%) and -1.54 (range: -44.0 to 27.3%), and RE_S was -0.39 and -0.97% for residual pulpwood cubic-foot volumes based on multiplying the proportion of residual pulpwood volume Equation 5 (height independent variables) or Equation 6 (height and diameter independent variables), respectively, times the pulpwood volume Equation 1 (Method 2).

For residual pulpwood volume, there was little difference between the relative errors for equations with height independent variables and equations with height and diameter independent variables. For Method 1, the relative errors were somewhat smaller for the equation with height and diameter

independent variables. However, for Method 2, the relative errors were larger for the equation with height and diameter independent variables. $\overline{RE}$ was smaller and RE_S was larger for residual pulpwood volume models based on Method 1 compared to Method 2.

One sample was taken in each of two aspen stands in Upper Michigan using 0.05-ac. circular plots. For each stand, DBH to the nearest 0.1 in. and merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter was measured for each tree with DBH $\geq$4.6 in. on each plot.

Five plots were selected in Stand 1. DBH varied from 5.2 to 11.4 in., and merchantable height varied from 2 to 6 sticks. The average number of trees per plot was 9.6 (range: 6 to 14).

Five plots were selected in Stand 2. DBH varied from 5.6 to 14.6 in., and merchantable height varied from from 1 to 6 sticks. The average number of trees per plot was 14.6 (range: 8 to 26).

Volume in rough cords was estimated for both stands using the new pulpwood prediction Equation 1 (Table 4), Table 6 of Gevorkiantz and Olsen, and Table 1 of Ek and Droessler.

Results for the two samples are shown in Table 8. For Stand 1, our new Equation 1 yielded estimates 3.0 and 15.9% higher than Table 6 of Gevorkiantz and Olsen and Table 1 of Ek and Droessler, respectively. For Stand 2, our new Equation 1 yielded estimates 1.1 and 17.3% higher than Table 6 of Gevorkiantz and Olsen and Table 1 of Ek and Droessler, respectively.

As expected, the new pulpwood volume equation yielded estimates somewhat higher than Table 6 of Gevorkiantz and Olsen and considerably higher than Table 1 of Ek and Droessler. In comparing these results, remember that we used cubic-foot per rough cord values varying from 73 to 80 while Gevorkiantz and Olsen and Ek and Droessler used 79. Also, our approximate 3.6-in. minimum top

Table 8. Volume estimates in rough cords per acre for the samples taken from 2 stands in the Upper Peninsula based on our Equation 1 (Table 4), Table 6 of Gevorkiantz and Olsen, and Table 1 of Ek and Droessler.

Stand	Equation 1	Gevorkiantz and Olsen (Table 6)	Ek and Droessler (Table 1)
1	24.1	23.4	20.8
2	47.4	46.9	40.4

diameter is different than the variable top diameters used by Gevorkiantz and Olsen and the constant 4-in. top diameter used by Ek and Droessler.

Guidelines for Users

The rough cord volumes obtained using the new pulpwood volume equation (Table 4) will yield, in general, per acre estimates from 5% lower to 8% higher and from 12-18% higher than volume estimates from Gevorkiantz and Olsen (Table 6) and Ek and Droessler (Table 1), depending on stand DBH.

Validation results in terms of relative errors indicated that there were mixed, but relatively small, differences between the residual pulpwood volume equations based on height (Equations 3 and 5) and height and diameter (Equations 4 and 6) independent variables. For Method 1 (residual pulpwood volume equation), the relative errors were somewhat smaller for the equation with height and diameter independent variables, while the reverse was true for Method 2 (proportion of residual pulpwood volume equation times the pulpwood volume equation). Thus, either equation of either method would yield adequate results for almost all situations. A strong argument can be made for Method 2. The use of Method 2 to obtain residual pulpwood volume followed by determining sawtimber volume as the difference between pulpwood volume and residual pulpwood volume yields a compatible approach to total merchantable volume estimation. The use

of the sawtimber volume and the Method 1 residual pulpwood volume equations would not yield the total volume obtained by the pulpwood volume equation. However, our results show very little difference between these two approaches to estimate total volume from sawtimber and residual pulpwood volume estimates.

It should be noted that Method 1 based on the equation with height independent variables can be simplified considerably without sacrificing much accuracy. The residual cubic-foot (Table 5) and rough cord (Table 6) volumes are relatively constant over the number of sawtimber sticks for a given number of residual pulpwood sticks. Cubic-foot (rough cord) volume values of 2.5 (0.032), 4.1 (0.053), 5.7 (0.073), 7.3 (0.093), 8.8 (0.113), 10.4 (0.133), 12.0 (0.153), and 13.6 (0.174) would yield quick, accurate residual volume values for trees with 1, 2, 3, 4, 5, 6, 7, and 8 residual pulpwood sticks, respectively.

We recommend the use of the following cubic-foot volume equations for most cruising situations:

1. Pulpwood volume - Equation 1 (Table 2, rough cord volume-Table 4).
2. Sawtimber volume - Equation 2 (Table 3).
3. Sawtimber and residual pulpwood volumes - Method 2 (height independent variables).
    A. Residual pulpwood cubic-foot volume - Equation 5 x Equation 1.
    B. Residual pulpwood rough cord volumes - Convert cubic-foot volumes to rough cord volumes.
    C. Sawtimber cubic-foot volume - subtract the product of Equation 5 and Equation 1 from Equation 1.

For those situations where somewhat more accuracy is needed, we recommend use of the following cubic-foot residual volume approach using height and DBH independent variables:

1. Sawtimber and residual pulpwood volumes - Method 2.

   A. Residual pulpwood cubic-foot volume (Equation 6 x Equation 1).

   B. Sawtimber cubic-foot volume (subtract product of Equation 6 and Equation 1 from Equation 1).

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Appendix

Table 2. Volume table showing pulpwood cu.ft. volume for various combinations of DBH and merchantable height in sticks to an approximate 3.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
5	1.1	1.8	2.5						
6	1.5	2.5	3.4	4.3					
7	1.9	3.2	4.5	5.7	6.8				
8	2.4	4.1	5.7	7.2	8.7	10.1			
9	2.9	5.0	7.1	9.0	10.8	12.6	14.3		
10	3.5	6.1	8.5	10.9	13.1	15.3	17.4		
11		7.3	10.2	12.9	15.6	18.2	20.8	23.2	
12		8.5	11.9	15.2	18.3	21.4	24.4	27.3	
13		9.8	13.8	17.6	21.3	24.8	28.3	31.7	
14		11.3	15.9	20.2	24.4	28.5	32.5	36.4	
15			18.0	23.0	27.8	32.4	36.9	41.4	45.7
16			20.3	25.9	31.3	36.5	41.6	46.7	51.6
17			22.7	29.0	35.0	40.9	46.6	52.2	57.7
18			25.2	32.2	39.0	45.5	51.9	58.1	64.2
19			27.9	35.6	43.1	50.3	57.4	64.3	71.1
20			30.7	39.2	47.4	55.4	63.1	70.7	78.2
21				42.9	51.9	60.6	69.1	77.5	85.6
22				46.8	56.6	66.1	75.4	84.5	93.4
23				50.9	61.5	71.8	81.9	91.8	101.5
24				55.1	66.6	77.8	88.7	99.4	109.9
25				59.4	71.9	83.9	95.7	107.3	118.6

Table 3. Volume table showing sawtimber cu.ft. volume for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
9	3.5	6.3	9.1	11.8	14.5				
10	4.0	7.4	10.6	13.8	17.0	20.1			
11	4.6	8.5	12.3	16.0	19.6	23.2			
12	5.2	9.7	14.0	18.3	22.5	26.6	30.7		
13	5.9	11.0	15.9	20.7	25.4	30.1	34.7	39.3	
14	6.6	12.3	17.8	23.2	28.5	33.7	38.9	44.1	
15	7.3	13.6	19.8	25.8	31.7	37.5	43.3	49.1	54.8
16		15.0	21.8	28.5	35.0	41.5	47.9	54.3	60.6
17		16.5	24.0	31.3	38.5	45.6	52.6	59.6	66.5
18		18.0	26.2	34.2	42.0	49.8	57.5	65.2	72.7
19		19.6	28.4	37.1	45.7	54.2	62.6	70.9	79.1
20		21.2	30.8	40.2	49.5	58.7	67.8	76.8	85.7
21			33.2	43.4	53.4	63.3	73.1	82.8	92.5
22			35.7	46.6	57.4	68.1	78.6	89.1	99.4
23			38.2	50.0	61.5	72.9	84.2	95.5	106.6
24			40.8	53.4	65.7	77.9	90.0	102.0	113.9
25			43.5	56.9	70.1	83.1	95.9	108.7	121.4

Table 4. Volume table showing rough cord volume for various combinations of DBH and merchantable height in sticks to an approximate 3.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
5	0.014	0.023	0.032						
6	0.019	0.032	0.044	0.054					
7	0.025	0.041	0.057	0.072	0.085				
8	0.031	0.052	0.073	0.091	0.108	0.125			
9	0.038	0.065	0.090	0.113	0.135	0.155	0.174		
10	0.046	0.078	0.109	0.137	0.163	0.188	0.211		
11		0.093	0.130	0.164	0.195	0.224	0.252	0.279	
12		0.109	0.153	0.192	0.229	0.263	0.296	0.328	
13		0.126	0.177	0.223	0.265	0.306	0.344	0.380	
14		0.144	0.203	0.255	0.304	0.351	0.395	0.437	
15			0.231	0.290	0.346	0.399	0.449	0.496	0.549
16			0.260	0.327	0.390	0.450	0.506	0.560	0.619
17			0.291	0.366	0.437	0.503	0.567	0.627	0.693
18			0.323	0.407	0.486	0.560	0.630	0.697	0.771
19			0.357	0.450	0.537	0.619	0.697	0.771	0.853
20			0.393	0.495	0.591	0.681	0.767	0.849	0.938
21				0.542	0.647	0.746	0.840	0.930	1.028
22				0.591	0.706	0.814	0.916	1.014	1.121
23				0.642	0.767	0.884	0.995	1.102	1.218
24				0.695	0.830	0.957	1.078	1.193	1.319
25				0.750	0.896	1.033	1.163	1.287	1.423

Table 5. Residual pulpwood cu.ft. volume for various numbers of sawtimber and residual pulpwood sticks based on Equation 3.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	2.79	4.36	5.94	7.51	9.09	10.66	12.24	13.82
2	2.66	4.23	5.81	7.39	8.96	10.54	12.11	13.69
3	2.58	4.16	5.73	7.31	8.89	10.46	12.04	13.61
4	2.53	4.11	5.68	7.26	8.83	10.41	11.99	13.56
5	2.49	4.07	5.64	7.22	8.79	10.37	11.94	13.52
6	2.46	4.03	5.61	7.18	8.76	10.33	11.91	13.49
7	2.43	4.00	5.58	7.15	8.73	10.31	11.88	13.46
8	2.40	3.98	5.55	7.13	8.71	10.28	11.86	13.43

Table 6. Residual rough cord volume for various numbers of sawtimber and residual pulpwood sticks based on Equation 3.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.036	0.056	0.076	0.096	0.116	0.137	0.157	0.177
2	0.034	0.054	0.074	0.095	0.115	0.135	0.155	0.175
3	0.033	0.053	0.073	0.094	0.114	0.134	0.154	0.174
4	0.032	0.053	0.073	0.093	0.113	0.133	0.153	0.174
5	0.032	0.052	0.072	0.092	0.113	0.133	0.153	0.173
6	0.031	0.052	0.072	0.092	0.112	0.132	0.152	0.173
7	0.031	0.051	0.071	0.092	0.112	0.132	0.152	0.172
8	0.031	0.051	0.071	0.091	0.111	0.132	0.152	0.172

Table 7. Proportions of residual pulpwood cu.ft. volume in a sawtimber tree for various numbers of sawtimber and residual pulpwood sticks based on Equation 5.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.44	0.54	0.62	0.68	0.73	0.77	0.81	0.84
2	0.26	0.35	0.44	0.50	0.55	0.59	0.63	0.66
3	0.17	0.25	0.32	0.39	0.44	0.49	0.52	0.55
4	0.12	0.18	0.25	0.31	0.36	0.41	0.45	0.48
5	0.09	0.13	0.19	0.25	0.30	0.35	0.38	0.42
6	0.06	0.09	0.15	0.20	0.25	0.30	0.33	0.37
7	0.04	0.06	0.11	0.17	0.21	0.26	0.29	0.33
8	0.03	0.04	0.08	0.13	0.18	0.22	0.26	0.29