

Michigan Department of Natural Resources

ASPEN PRISM SAMPLING

This analysis was undertaken to determine the feasibility of using cumulative tally methods to obtain accurate estimates of combinations of pulpwood and sawlogs when prism sampling aspen in Michigan. The results from any cruise on MDNR Timber Sales should be consistent with results calculated with the Fowler/Hussain VBARs for aspen and red pine. For other species, results should be consistent with Carlson VBAR for pulp and MDNR VBAR for Sawlogs. This is according to the product standards adopted by the Division in 1987.

Cumulative tally of prism data is predicated on the assumption of linear increase in the Volume to Basal Area Ratio (VBAR) when related to the increase in height measurement. Though any VBAR relationship using cumulative tally will exhibit problems at the extremes of existing forest conditions, it is possible that such formulas will produce nearly identical results over a wide range of existing conditions. There can be no doubt that cumulative tally of prism data is the most efficient method of field data collection and input. Out of the 13 responses to the committee's request for input, a clear majority were using some form of cumulative tally.

The first step in the analysis was to apply a simple linear regression to the pulpwood volumes obtained by using the Height Independent VBAR formula prepared by Fowler and Hussain. Because trees with 2 through 7 sticks in height are by far and away the most prevalent for any given cruise, only these values were included in the regression. Table 1 indicates the results of this while Figure 1 graphically shows how successful this approximation is.

Table 1: Comparison of Volume to Basal Area results predicted by formulas prepared for individual and cumulative prism tally. Computed directly in Cubic Feet per Square Foot.

	Indiv VBAR	Cumm VBAR	Cum/ Ind	Regression Output for Individual Tally VBAR		
	1	5.7180	9.0984	159.1%	Constant	5.331301
	2	12.4088	12.8656	103.7%	Std Err of Y Est	0.353785
Total	3	16.8503	16.6327	98.7%	R Squared	0.997988
Number	4	20.7294	20.3998	98.4%	No. of Observations	6
of	5	24.3836	24.1669	99.1%	Degrees of Freedom	4
Merch.	6	27.9253	27.9341	100.0%		
Sticks	7	31.4028	31.7012	101.0%	X Coefficient(s)	3.767126
(PH)	8	34.8401	35.4683	101.8%	Std Err of Coef.	0.084571
	9	38.2506	39.2354	102.6%		
	10	41.6424	43.0026	103.3%		

$$\text{Pulp VBAR (ind)} = 9.1492 + 3.3168 * \text{PH} - 6.7480/\text{PH}$$

$$\text{Pulp VBAR (cum)} = 5.331301 + 3.767126 * \text{PH}$$

Figure 1.

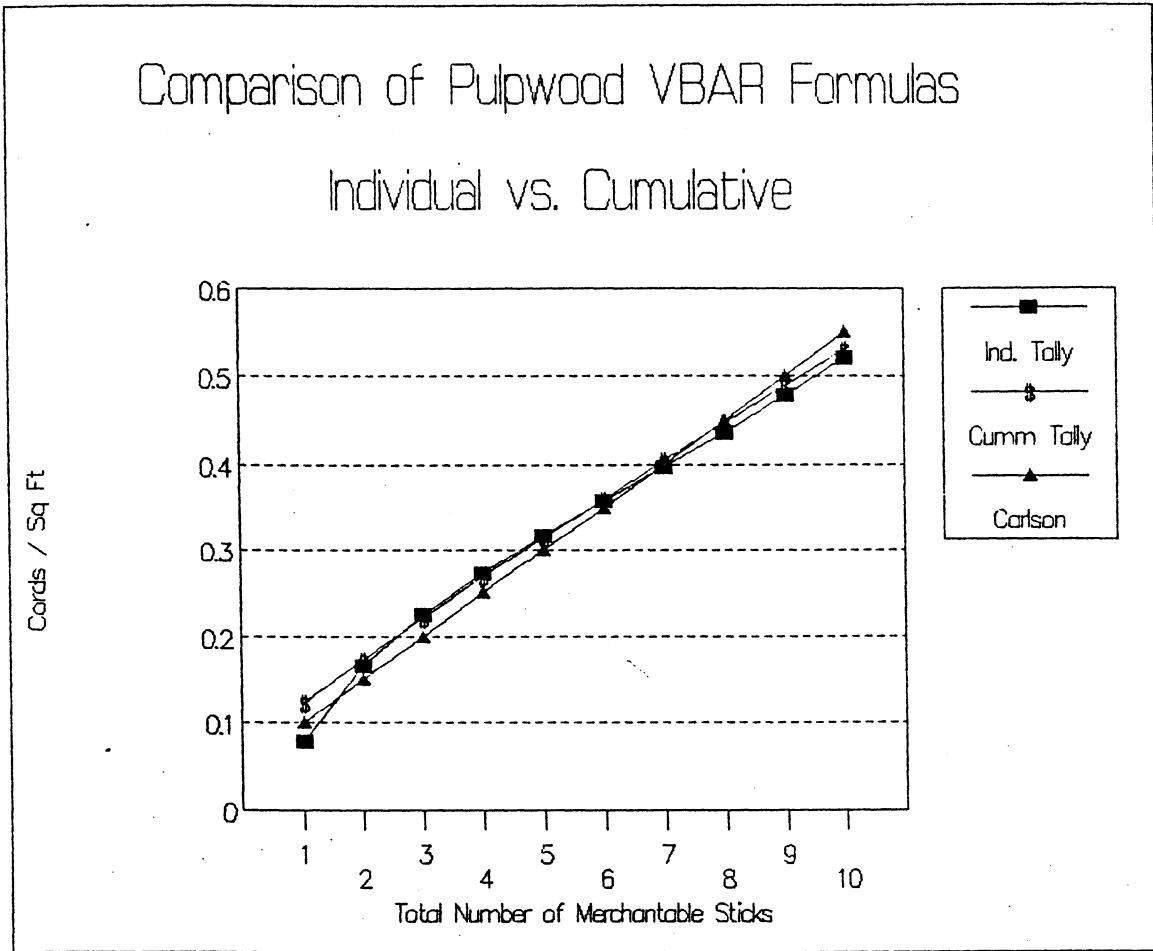


Table 2. Cubic Foot to Cord Conversion.

$$\text{Cords} = (.013697 - .00014 * \text{PH}) * \text{Cubic Feet}$$

	Cords per Cubic Ft	Actual cf to cd	Predicted cf to cd	Pred to Actual
Total	73.0	0.0137	0.0136	99.0%
Number	75.0	0.0133	0.0134	100.6%
of	75.0	0.0133	0.0133	99.6%
Merch.	76.0	0.0132	0.0131	99.8%
Sticks	77.0	0.0130	0.0130	100.1%
(PH)	78.0	0.0128	0.0129	100.3%
	79.0	0.0127	0.0127	100.5%
	80.0	0.0125	0.0126	100.6%
	80.0	0.0125	0.0124	99.5%
10	80.0	0.0125	0.0123	98.4%

Regression Output For
CF to Cord Conversion

Constant	0.013697
Std Err of Y Est	0.0001
R Squared	0.949885
No. of Observations	10
Degrees of Freedom	8
X Coefficient(s)	-0.00014
Std Err of Coef.	0.000011

Next, to make a comparison of the various VBAR formulas in use, the individual tally VBAR and cumulative tally VBAR volumes were converted to cords according to the conversion values stated by Fowler and Hussain. Table 2 indicates a possible formula for expediting the conversion for Aspen.

The comparison shown in Table 3 shows the MDNR VBAR offering the lowest volume per square foot of basal area in all but the 1 stick trees. Clearly the Cumulative VBAR will produce the highest estimate in stands with average aspen heights of less than 2.5 sticks. The assertion here is that very few merchantable aspen trees on any given sale are less than 2 sticks. Any stand with aspen averages that low would be the lowest volume and value stands we would encounter. For those cases, reversion to the Carlson VBAR could produce consistent results.

Table 3. Comparison of Pulpwood VBARS in Cords per Square Foot.

	Indiv VBAR	Cumm VBAR	MDNR VBAR	Carlson VBAR
1	0.078	0.123	0.086	0.100
2	0.165	0.173	0.144	0.150
Total	0.225	0.221	0.201	0.200
Number	0.273	0.268	0.251	0.250
of	0.317	0.315	0.296	0.300
Merch.	0.358	0.360	0.333	0.350
Sticks	0.398	0.404	0.368	0.400
(PH)	0.436	0.447	0.391	0.450
9	0.478	0.489		0.500
10	0.521	0.531		0.550

Table 4 lists the Sawlog VBARS calculated when both topwood and sawlogs are present in a tree. According to Fowler and Hussain the best estimate is derived by calculating the total volume in the tree and subtracting the topwood portion. This leaves you with the sawlog portion in cubic feet. On the other hand, effective cumulative tally requires that sawlog volumes be subtracted from total volumes to obtain pulp volumes. For purposes of this analysis, Table 4 includes only volumes for height combinations encountered among the 802 trees tallied for the VBAR research. A weighted average was calculated from these VBARS. These were compared to the volumes calculated by the Sawlog VBAR formula. The calculated values range little from the average VBAR. As a result, this average VBAR seems suitable for regression analysis for the purposes of considering cumulative tally of sawlogs and pulp.

Table 4: Volume to Basal Area results predicted by formulas prepared by Fowler/Hussain. Pulpwood VBAR - Residual VBAR in CuFt/SqFt

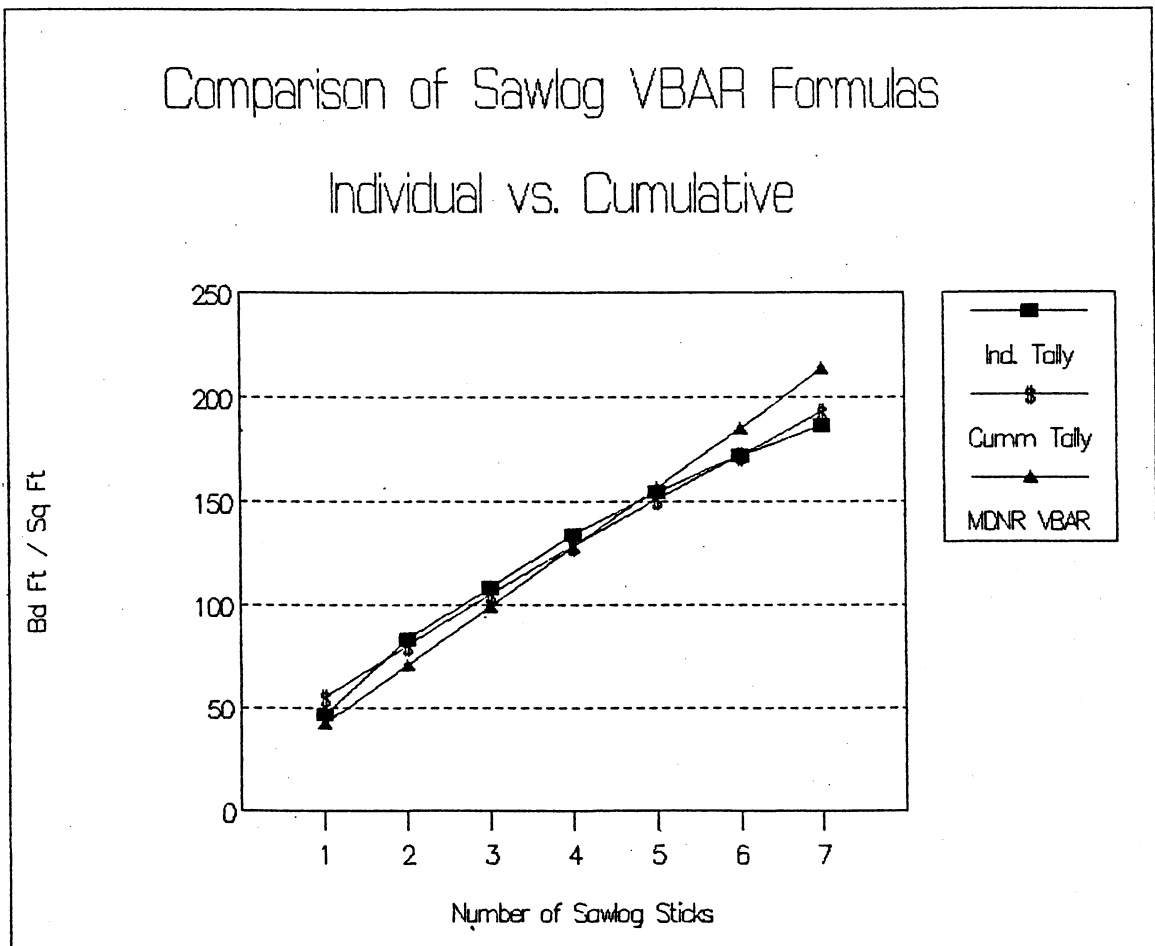
		Number of Topwood Sticks					Average	Sawlog VBAR
		1	2	3	4	5		
1		7.7163	7.8756	7.8257	7.6202	7.2684	7.6612	7.5485
Number	2	13.4109	13.7543	13.9692	14.1210	14.2191	13.8949	13.8803
Of	3	17.1303	18.3882	18.8538	19.1705	19.4372	18.5960	18.7782
Sawlog	4	21.4090	23.0179	23.6142	24.0135	24.3524	23.2814	23.3177
Sticks	5	25.4764	27.4437	28.1760	28.6549		27.4377	27.7138
	6	29.4163	31.7385	32.6074			31.2541	32.0382
	7	33.2739	35.9438				34.6088	36.3216

Table 5: Volume to Basal Area results predicted by formulas prepared for cumulative prism tally. Computed directly in CuFt/SqFt (Sawlog VBAR = 4.61876 + 4.442959 * No Of Sawlog Sticks)

	Ave Individ. VBAR	Cumm. VBAR	Cumm/ Indiv.	Regression Output:		
	1	7.6612	9.0617	118.3%	Constant	4.61876
	2	13.8949	13.5047	97.2%	Std Err of Y Est	0.992639
Number	3	18.5960	17.9476	96.5%	R Squared	0.991165
Of	4	23.2814	22.3906	96.2%	No. of Observations	7
Sawlog	5	27.4377	26.8336	97.8%	Degrees of Freedom	5
Sticks	6	31.2541	31.2765	100.1%	X Coefficient(s)	4.442959
	7	34.6088	35.7195	103.2%	Std Err of Coef.	0.187591

Table 5 shows how closely a simple linear regression can approximate the values obtained from Table 4. Figure 2 graphically demonstrates the close fit.

Figure 2.



Fowler and Hussain did not address Cubic Foot to Board Foot conversion in their research on aspen. Such a conversion formula is addressed in a paper entitled "Tree Volume and Biomass Equations for the Lake States" prepared by Jerold T. Hahn in 1984. He developed a formula using Robert Stone's data using DBH, height in Feet (H), and a representation of the top diameter (T).

$$T = (1.00001 - \text{Top DOB/DBH})$$

$$\text{Bd Ft} = 17.7488 + 7.3846 * \text{Cu Ft} - 2.3523 * \text{DBH} - 0.89945 * \text{H} + 2.0726 * \text{T}$$

Of course, DBH is not available from our cruises while this formula demands it. My solution was to solve for DBH in the height and diameter independent VBAR formulas using the height independent VBARS from the Aspen research. Table 6 lists the VBARS used the formula, the resulting DBHs, and the Board Foot VBARS calculated with the formula listed above.

Table 6. Conversion of Cubic Ft VBARS to Board Foot VBARS by calculating DBH and using Hahn's Formula.

	Ind VBAR CF/SqFt	Derived DBH	Ind VBAR BF/SqFt
1	7.66	9.03	46.22
2	13.89	9.93	83.10
Number 3	18.60	11.19	107.83
Of 4	23.28	11.98	133.47
Sawlog 5	27.44	13.13	154.37
Sticks 6	31.25	14.46	172.35
7	34.61	16.08	186.23

Table 7 compares the Board Foot and Cubic Foot values for these data points and shows the results of a regression of this conversion data.

Table 7. Cubic Foot to Board Foot Conversion

$$\text{Board Feet (Int)} = (6.160955 - .10929 * \text{SH}) * \text{Cubic Feet}$$

	Bd Ft VBAR	Cu Ft VBAR	BF / CF	Predicted BF / CF	Regression Output:	
1	46.22	7.66	6.03	6.05	Constant	6.160955
2	83.10	13.89	5.98	5.94	Std Err of Y Est	0.026665
Number 3	107.83	18.60	5.80	5.83	R Squared	0.989481
Of 4	133.47	23.28	5.73	5.72	No. of Observations	7
Sawlog 5	154.37	27.44	5.63	5.61	Degrees of Freedom	5
Sticks 6	172.35	31.25	5.51	5.51	X Coefficient(s)	-0.10929
7	186.23	34.61	5.38	5.40	Std Err of Coef.	0.005039

Table 8 compares the results of applying this conversion to the Aspen VBARS to the other commonly used Sawlog VBARS. The figures compare well within reason, lending support to validity of this conversion formula.

Data reduction with this cumulative tally data is as follows:

- 1) Sum up each data item (TT, TS, LT, LS)
- 2) Total CF/Ac = $[(5.3313 * \sum TT + 3.7671 * \sum TS) * BAF] / \text{Number of Plots}$
- 3) Sawlog CF/Ac = $[(4.6188 * \sum LT + 4.4430 * \sum LS) * BAF] / \text{Number of Plots}$
- 4) Pulp CF/Ac = Total CF/Ac - Sawlog CF/Ac

Conversions:

- 5) Pulp Cords/Ac = $[\text{.013697} - \text{.00014} * (\sum TS - \sum LS) / \sum TT] * \text{Pulp CF/Ac}$
- 6) Sawlog MBF/Ac = $[(\text{6.088} - \text{.0712} * \sum LS / \sum LT) * \text{Sawlog CF/Ac}] / 1000$