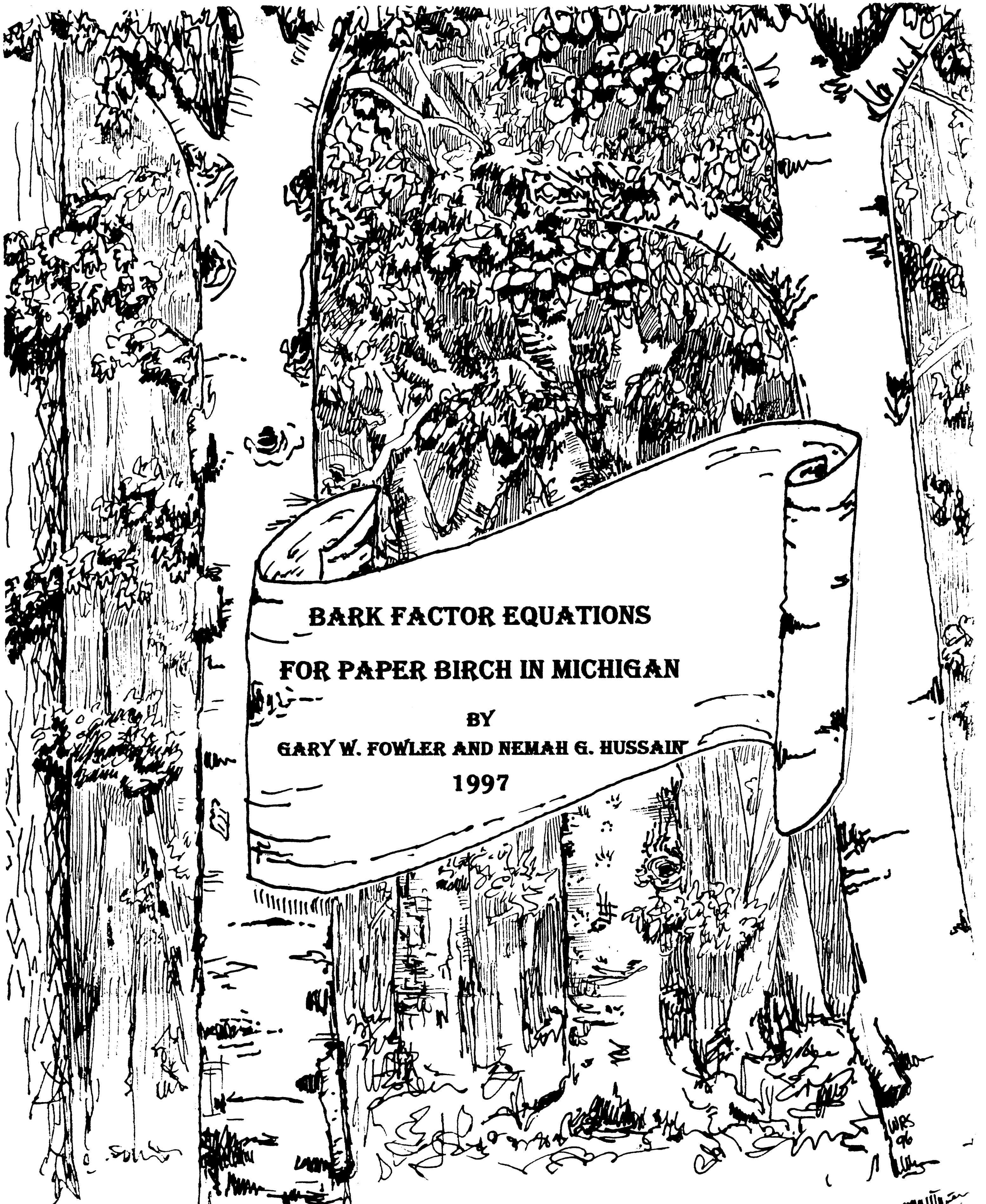

MDNR FOREST MANAGEMENT DIVISION

INFORMATION LEAFLET 1-97

FORESTRY INFORMATION LEAFLET
FOREST MANAGEMENT DIVISION
MICHIGAN
DEPARTMENT OF NATURAL RESOURCES

NUMBER - 1-97

SUBJECT - BARK FACTORS

DATE - 20 July 97

TITLE - BARK FACTOR EQUATIONS FOR PAPER BIRCH IN MICHIGAN

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BACKGROUND

Bark factor (BF) is the ratio of diameter inside bark (DIB) to diameter outside bark (DOB) at a given tree height. Even though bark factor does increase with height for many species, a constant bark factor, usually determined at breast height, has been assumed, in many cases, for all tree heights for many species. Thus, the use of a constant bark factor for all tree heights will usually lead to underestimates of most tree and log solid wood volumes and overestimates of bark volume for many species. Bark factor equations have been developed for aspen (Fowler and Hussain 1987b, Fowler 1991), jack pine (Fowler and Hussain 1991, Fowler 1993), and red pine (Fowler and Hussain 1987a, Fowler and Damschroder 1988) in Michigan where bark factor was regressed on tree height. In all cases, there was a very strong relationship between bark factor and tree height.

PURPOSE

The purpose of this paper is to present bark factor prediction equations for paper birch in Michigan and show how the prediction equations may be used.

METHODS AND MATERIALS

As part of a larger study to develop new volume equations for hardwoods in Michigan, felled tree measurements were made on a total of 84 paper birch trees (44 trees from a stand in the Copper Country State Forest, and 39 trees and one tree from two stands, respectively, from the Escanaba River State Forest) during May-August, 1995. DIB and DOB were measured to the nearest 0.01 in. at stump height, which varied from 3-28 in., at the top of each 8.3-ft. bolt (100-in. stick) cut out of the stem of each tree to an approximate 3.6-in. diameter top limit (i.e., stemwood), and at the bottom and top of each 8.3 ft. bolt cut out of any limbs and top forks of each tree to an approximate 3.6-in. diameter top limit (i.e., topwood). Diameter at breast height (DBH) was measured to the nearest 0.1 in., and bark thickness at DBH height was measured to the nearest 0.01 in. DBH height was 4.5 ft. from the ground except for trees that forked below 4.5′ where DBH height was approximately 4.5′ above the fork. DBH varied from 3.9-16.0 in. for the data set of 84 trees.

Stemwood

The prediction data set included 45 trees (44 trees from the Copper Country stand and the Escanaba River stand with one tree), where DBH varied from 3.9 to 14.4 in. and merchantable height (MH), the number of 100-in. sticks of stemwood to an approximate 3.6-in. top diameter limit, varied from 1-6 sticks. This yielded a data set of 225 bark factor measurements where BF varied from 0.958 to 0.996, tree height to measurement (TH) varied from 0.25 to 51.08 ft., and DOB at TH varied from 3.26 to 16.40 in.

The validation data set included the 39 trees from the second Escanaba River stand, where DBH varied from 4.8 to 16.0 in. and MH varied from 2-7 sticks. This yielded a data set of 272 bark factor measurements where BF varied from 0.959 to 0.995, TH varied from 0.42 to 59.08 ft., and DOB at TH varied from 3.14 to 17.71 in.

Topwood

Twelve of the total of 84 paper birch trees had topwood sticks. For these trees, DBH varied from 7.8 to 16.0 in., MH varied from 3-6 sticks, and the number of topwood sticks (TS) varied from 1-11 with a mean of 3.6 sticks.

The prediction data set included 8 trees from the Escanaba River stand with a total of 39 trees, where DBH varied from 9.2 to 16.0 in., MH varied from 3-6 sticks, and TS varied from 2-11 with an average of 4.6 sticks. This yielded a data set of 53 bark factor measurements where BF varied from 0.960 to 0.991 and DOB at the point where BF was determined varied from 3.00 to 11.18 in.

The validation data set included four trees (three trees from the Copper Country stand and the Escanaba River stand with one tree), where DBH varied from 8.4 to 14.4 in., MH varied from 4 to 6 sticks and TS varied from 1 to 2 sticks with an average of 1.5 sticks. This yielded a data set of 11 bark factor measurements where BF varied from 0.951 to 0.987 and DOB varied from 3.50 to 7.44 in.

Prediction equations

All prediction equations were developed using multiple linear regression. For stemwood, BF was regressed on DOB and TH using the prediction data set of 225 cases. For topwood, BF was regressed on DOB using the prediction data set of 53 cases. The stemwood and topwood prediction equations were validated on the validation data sets of 272 and 11 cases, respectively. The final prediction models for both stemwood and topwood were developed using the respective prediction and validation data sets pooled. The final stemwood and topwood prediction data sets were then compared.

RESULTS

The best prediction equations, based on simplicity, meeting the assumptions of normality and homogeneity, and having among the smallest standard errors of the estimate ($s_{y \bullet x}$) and the largest coefficients of determination (R^2), were

Stemwood

(1) $\hat{BF} = 0.971906 + 0.001280 \bullet DOB$

$R^2 = 0.232 \quad s_{y \bullet x} = 0.006188$

(2) $\hat{BF} = 0.983365 - 0.002153 \bullet \ln(TH)$

$R^2 = 0.176 \quad s_{y \bullet x} = 0.009532$

(3) $\hat{BF} = 0.976201 + 0.000997 \bullet DOB - 0.001599 \ln(TH)$

$R^2 = 0.260 \quad s_{y \bullet x} = 0.006210$

Prediction Equations 1 and 2 yield the following estimated bark factors.

Prediction Equation 1		Prediction Equation 2	
DOB (in.)	$\hat{BF}$	TH (ft.)	$\hat{BF}$
4.0	0.977	0.25	0.986
5.0	0.978	0.5	0.985
6.0	0.980	1.0	0.983
7.0	0.981	2.0	0.982
8.0	0.982	4.5	0.980
9.0	0.983	8.5	0.979
10.0	0.985	17.0	0.977
11.0	0.986	25.5	0.976
12.0	0.987	34.0	0.976
13.0	0.989	42.5	0.975
14.0	0.990	51.0	0.975
15.0	0.991	59.5	0.975
16.0	0.992	68.00	0.974

The predicted BF based on Equation 1 varies from 0.986 for TH=0.25 ft. to 0.974 for TH=68.00 ft., yielding a range of 0.012. The predicted BF based on Equation 2 varies from 0.977 for DOB=4.0 in. to 0.992 for DOB=16.0 in., yielding a range of 0.015. Because of these small ranges and the low R^2 values of the prediction equations, you might argue that the mean bark factor of the 225 bark factor measurements yields an adequate prediction model.

$$(4) \quad \hat{BF} = \overline{BF} = \sum_{i=1}^{225} BF_i / 225 = 0.979$$

$$s_y = 0.007186$$

Topwood

$$(5) \quad \hat{BF} = 0.96820 + 0.001561 \bullet DOB$$

$$R^2 = 0.186 \quad s_{y \bullet x} = 0.006469$$

This prediction equation yields the following estimated bark factors.

DOB (in.)	$\hat{BF}$	DOB (in.)	$\hat{BF}$
3.0	0.973	10.0	0.984
4.0	0.974	11.0	0.985
5.0	0.976	12.0	0.987
6.0	0.978	13.0	0.988
7.0	0.979	14.0	0.990
8.0	0.981	15.0	0.992
9.0	0.982	16.0	0.993

The predicted BF based on Equation 5 varies from 0.973 for DOB=3.0 in. to 0.993 for DOB=16.0 in., yielding a range of 0.020. Because of this small range and the low R^2 value of the prediction equation, you might argue that the mean bark factor of the 53 bark factor measurements yields an adequate prediction model.

(6) $\hat{BF} = \overline{BF} = \sum_{i=1}^{53} BF_i/53 = 0.978$

$s_y = 0.007100$

The mean bark factors for stemwood and topwood differ by only 0.001.

VALIDATION

The four prediction equations for stemwood yielded accurate predictions for the validation data set of 272 bark factor measurements. The average relative error was –0.04, –0.31, –0.16, and –0.19% for Equations 1, 2, 3, and 4, respectively. Relative error is the difference between predicted and actual bark factor divided by actual bark factor times 100. All predictions were between –1.42 and 2.10, –1.83 and 1.74, –1.62 and 1.85, and –1.53 and 2.16% for Equations 1, 2, 3, and 4, respectively. Equation 4 based on the mean bark factor compared favorably to the other 3 equations, overestimating for DOB < approximately 5.5 in. and TH > approximately 8.5 ft., and underestimating for DOB > approximately 5.5 in. and TH < approximately 8.5 ft. (see tables on page 4).

The two prediction equations for topwood also yielded accurate predictions for the validation data set of 11 bark factor measurements. The average relative error was 0.40 and 0.60% for Equations 5 and 6, respectively. All predictions were between –0.75 and 2.42, and -0.95 and 2.70% for Equations 5 and 6, respectively. Equation 6 based on the mean bark factor compared favorably to Equation 5, overestimating for DOB < approximately 6.0 in. and underestimating for DOB>approximately 6.0 in. (see table on page 5).

Pooled prediction equations

The prediction and validation data sets were pooled separately for stemwood and topwood, yielding pooled data sets consisting of 497 bark factor measurements from 84 trees and 64 bark factor measurements from 12 trees, respectively.

The final pooled equations for stemwood are:

(7) $\hat{BF} = 0.971380 + 0.001325 \bullet DOB$

$R^2 = 0.219 \quad s_{y \bullet x} = 0.006343$

(8) $\hat{BF} = 0.984990 - 0.002126 \bullet \ln(TH)$

$R^2 = 0.166 \quad s_{y \bullet x} = 0.006553$

(9) $\hat{BF} = 0.976408 + 0.001037 \bullet DOB - 0.001406 \ln(TH)$

$R^2 = 0.281 \quad s_{y \bullet x} = 0.006091$

Prediction Equations 7 and 8 yield the following estimated bark factors.

Prediction Equation 7		Prediction Equation 8	
DOB (in.)	$\hat{BF}$	TH (ft.)	$\hat{BF}$
4.0	0.977	0.25	0.988
5.0	0.978	0.5	0.986
6.0	0.979	1.0	0.985
7.0	0.981	2.0	0.984
8.0	0.982	4.5	0.982
9.0	0.983	8.5	0.980
10.0	0.985	17.0	0.979
11.0	0.986	25.5	0.978
12.0	0.987	34.0	0.977
13.0	0.989	42.5	0.977
14.0	0.990	51.0	0.977
15.0	0.991	59.5	0.976
16.0	0.993	68.0	0.976

The predicted BF based on Equation 7 varies from 0.988 for TH=0.25 ft. to 0.976 for TH=68.0 ft., yielding a range of 0.012. The predicted BF based on Equation 8 varies from 0.977 for DOB=4 in. to 0.993 for DOB=16.0 in., yielding a range of 0.016. Because of these small ranges and the low R^2 values of the prediction equations, you might argue that the mean bark factor of the 497 bark factor measurements yields an adequate prediction model.

(10) $\hat{BF} = \overline{BF} = \sum_{i=1}^{497} BF_i/497 = 0.980$

$s_y = 0.007169$

See tables on page 7 to find where Equation 10 over- and underestimates related to Equations 7 and 8.

The final pooled equations for topwood are:

(11) $\hat{BF} = 0.964872 + 0.002010 \bullet DOB$

$R^2 = 0.281 \quad s_{y \bullet x} = 0.007$

This prediction equation yields the following estimated bark factors for DOB.

DOB (in.)	$\hat{BF}$	DOB (in.)	$\hat{BF}$
3.0	0.971	10.0	0.985
4.0	0.973	11.0	0.987
5.0	0.975	12.0	0.989
6.0	0.977	13.0	0.991
7.0	0.979	14.0	0.993
8.0	0.981	15.0	0.995
9.0	0.983	16.0	0.997

The predicted bark factor based on Equation 11 varies from 0.971 for DOB=3 in. to 0.997 for DOB=16.0 in., yielding a range of 0.026. Because of this small range and the low R^2 value of the prediction equation, you might argue that the mean bark factor of the 64 bark factor measurements yields an adequate prediction model.

(12) $\hat{BF} = \overline{BF} = \sum_{i=1}^{64} BF_i/64 = 0.977$

$s_y = 0.008160$

See above table to find where Equation 12 over- and underestimates related to Equation 11.

Even though Equations 7 and 11 are significantly different (F-test for equal variances, $p \approx 0.10$; F-test for equal slopes, p=0.124; F-test for equal intercepts, p=0.006), and Equations 10 and 12 are significantly different (Bartlett's χ^2-test for equal variances, p=0.158; t-test for equal means, p=0.0004), you might pool each of the two sets separately because of simplicity and the relatively small difference between the two equations in each set. This yields the following equations for stemwood and topwood pooled.

$$(13) \quad \hat{BF} = 0.970591 + 0.001402 \bullet DOB$$

$$R^2 = 0.222 \quad s_{y \bullet x} = 0.006500$$

$$(14) \quad \hat{BF} = \overline{BF} = \sum_{i=1}^{561} BF_i/561 = 0.980$$

Prediction Equation 13 yields the following estimated bark factors.

DOB (in.)	$\hat{BF}$	DOB (in.)	$\hat{BF}$
3.0	0.975	10.0	0.985
4.0	0.976	11.0	0.986
5.0	0.978	12.0	0.987
6.0	0.979	13.0	0.989
7.0	0.980	14.0	0.900
8.0	0.982	15.0	0.992
9.0	0.983	16.0	0.993

The predicted BF based on Equation 13 varies from 0.975 for DOB=3.0 in. to 0.993 for DOB=16.0 in., yielding a range of 0.018. Because of this small range and the low R^2 value of the prediction equation, you might argue that the mean bark factor of the 561 bark factor measurements (Equation 14) yields an adequate model. See the above table to find where Equation 14 over- and underestimates related to Equation 13.

For the pooled stemwood data set (n=497), average bark thickness (BT) was 0.0622 in. (min.=0.0175, max.=0.1375). BT was not related to TH (r=0.008, p=0.863), but it was positively related to DOB (r=0.403, P<.000005). Average BT for the 4, 8, 12, and 16 one-inch DOB classes was 0.0484, 0.0732, 0.0802, and 0.0962 in., respectively.

For the pooled topwood data set (n=64), average BT was 0.0654 in. (min.=0.0220, max.=0.0950). BT was positively related to DOB (r=0.380, p=0.002). Average BT for the 3, 5, 7, and 9 one-inch DOB classes was 0.0545, 0.0702, 0.0700, and 0.0762, respectively.

Fowler (1993) showed that while there were significant species differences between bark factor equations for aspen, jack pine, and red pine, there was a very strong relationship between bark factor and tree height for each species (i.e., R^2>0.97 for each species). BF was a function of TH and ln TH, showing that BF increased with TH to some maximum and then decreased for larger TH's with the steepness of the decrease depending on the species. For all three species, BF was not strongly related to DBH or DOB at a given TH. This study shows that for paper birch, BF is much more variable than for aspen, red pine, and jack pine. The prediction equations, even though they are significant, are not very strong. For stemwood, BF decreased somewhat with TH (R^2=0.166) and increased somewhat with DOB (R^2=0.219). The relationship of BF with DOB was somewhat stronger than the relationship with TH. For topwood, BF increased somewhat with DOB (R^2=0.218).

GUIDELINES FOR USERS

We recommend use of the following equations for paper birch when accurate estimates of bark factors are desired:

- Stemwood

  (1) $\hat{BF} = 0.971380 + 0.001325 \bullet DOB$

  (2) $\hat{BF} = 0.9984990 - 0.002126 \bullet \ln(TH)$

  Use Equation 1 if DOB is measured. Use Equation 2 when only TH is measured.

- Topwood

  (3) $\hat{BF} = 0.964872 + 0.002010 \bullet DOB$

The equation for stemwood and topwood pooled could be used if DOB is measured with little loss in accuracy, especially for stemwood.

(4) $\hat{BF} = 0.970591 + 0.001402 \bullet DOB$

For adequate accuracy in most situations, the following constants could be used for bark factors.

DOB (in.)	Stemwood	Topwood	Pooled
DOB≤6.0	0.978	0.974	0.978
6.0>DOB≤9.0	0.982	0.981	0.982
9.0>DOB≤12.0	0.986	0.987	0.986
DOB>12.0	0.990	0.994	0.990

TH (ft.)	Stemwood
TH≤0.05	0.987
0.5<TH≤2.0	0.984
2.0<TH≤8.5	0.981
8.5<TH≤34.0	0.978
TH>34.0	0.977

The following constants for bark factor could be used for simplicity with good approximate results, especially for a large number of sticks.

Stemwood	Topwood	Stemwood and Topwood
0.980	0.977	0.980

The prediction equations can be used to estimate BF at any DOB or tree height. Since BF=DIB/DOB, DIB can be estimated as $\hat{DIB} = \hat{BF} \bullet DOB$ and DOB can be estimated as $\hat{DOB} = DIB / \hat{BF}$. Past DOB and DOB growth can be determined from past DIB growth as follows:

$$\text{Past DOB Growth} = \text{Past DIB Growth} / \hat{BF}$$

and

$$\text{Past DOB} = \text{Present DOB} - \text{Past DOB Growth}$$

where past DIB growth might be obtained with an increment borer.

Specific uses of the prediction equations include: (1) estimation of the solid wood and bark volume of standing trees, (2) estimation of bark volume, or peeled volume from unpeeled volume, of felled tree sections, (3) growth studies, and (4) estimating tree form (e.g., Girard Form Class).

See Husch et al. (1982) for a detailed discussion on bark factors.

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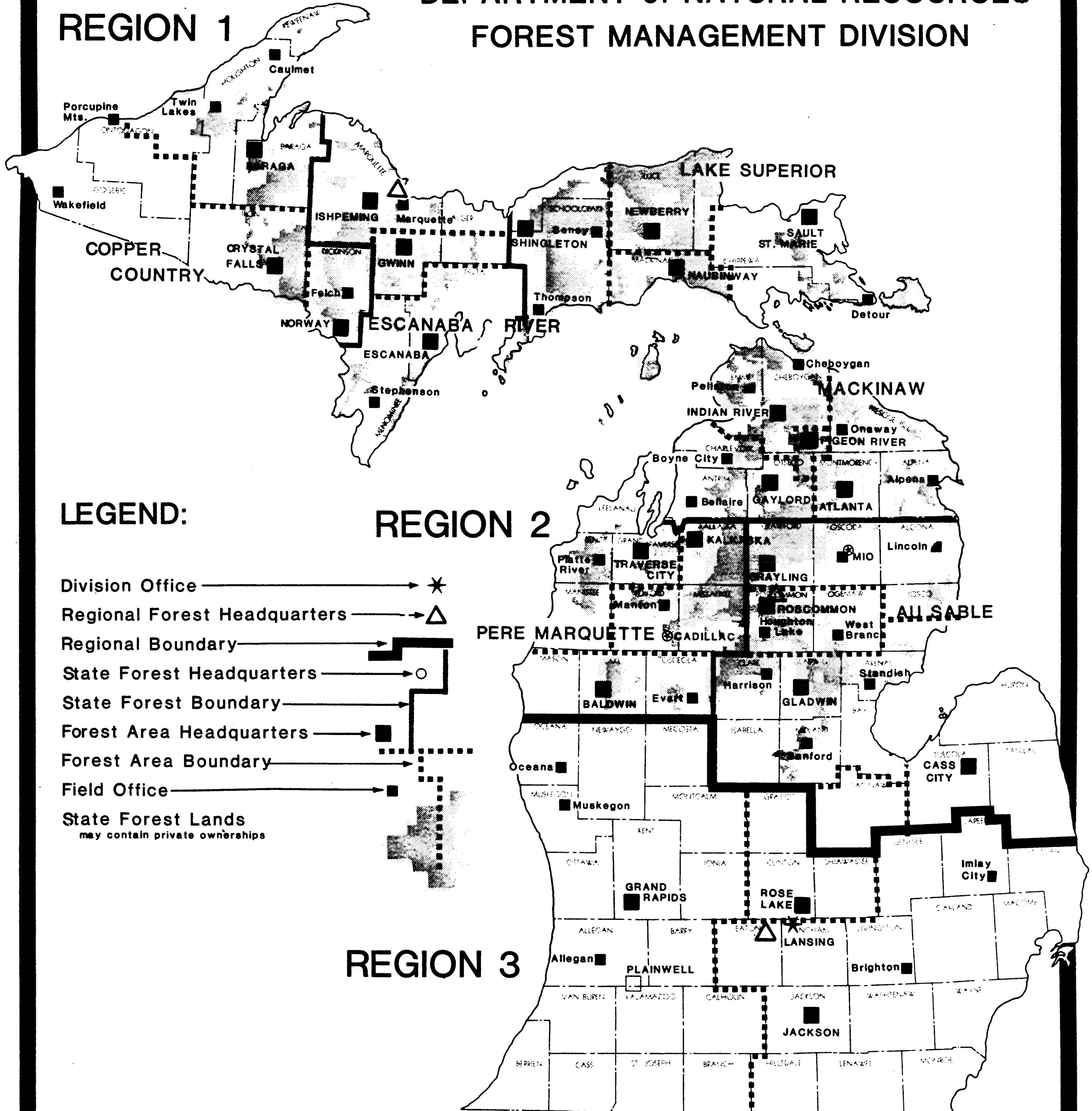
MICHIGAN'S STATE FOREST SYSTEM

DEPARTMENT of NATURAL RESOURCES
FOREST MANAGEMENT DIVISION



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