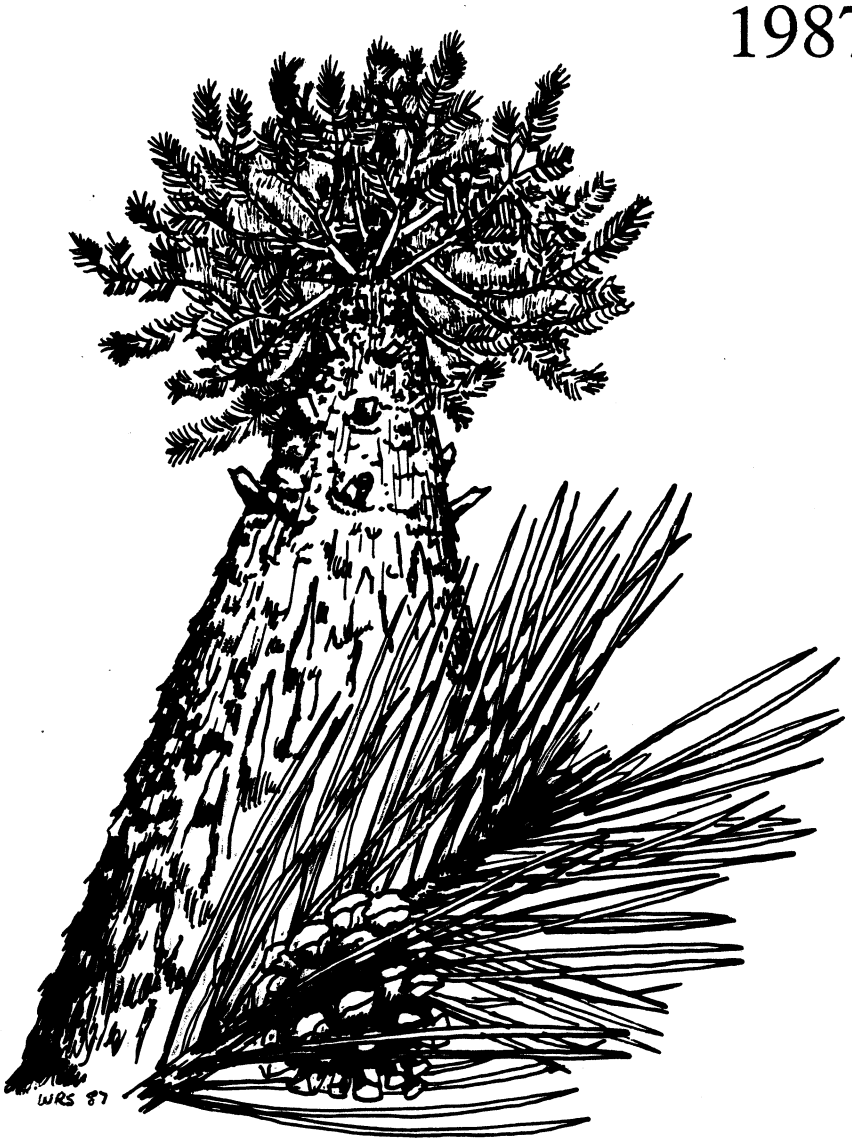
VOLUME-BASAL AREA RATIO EQUATIONS FOR RED PINE IN MICHIGAN

1987



By: Gary W. Fowler
Nemah G. Hussain

FOREST MANAGEMENT DIVISION
MICHIGAN DEPARTMENT OF NATURAL RESOURCES

SCHOOL OF NATURAL RESOURCES
UNIVERSITY OF MICHIGAN

Volume-Basal Area Ratio Equations for Red Pine in Michigan

by

Gary W. Fowler and Nemah G. Hussain

1987

Forest Management Division
Michigan Department of Natural Resources
and
School of Natural Resources
The University of Michigan

Management Summary

New total, pulpwood, sawtimber, and residual pulpwood cubic-foot volume-basal area ratio (VBAR in cu.ft./sq.ft.) equations were developed for red pine in Michigan. Data used to develop these equations were collected from 27 red pine stands in Michigan (16 and 11 stands from the Upper and Lower Peninsulas, respectively).

VBAR equations using diameter at breast height (DBH) and height independent variables yielded higher coefficients of determination (R^2), lower standard errors of estimate ($s_{y \cdot x}$), and more accuracy than VBAR equations using only height independent variables. However, an examination of prediction relative errors indicated that the height only VBAR equations are more than adequate for most situations. The new pulpwood VBAR equation based on height independent variables will usually yield volume estimates from 4 to 10% higher than estimates based on the VBARs presently used by the Michigan Department of Natural Resources.

We recommend the use of the following VBAR equations in most cruising situations for red pine:

1. Total VBAR

$$\hat{VBAR}_T = 12.9975+0.3375 \cdot TH-285.1908 \cdot \frac{1}{TH}$$

2. Pulpwood VBAR

$$\hat{VBAR}_P = 11.8783+3.2110 \cdot PH-8.4061 \cdot \frac{1}{PH}$$

3. Sawtimber VBAR

$$\hat{VBAR}_S = 11.4395+3.5298 \cdot SH-7.1381 \cdot \frac{1}{SH}$$

4. Residual pulpwood VBAR

$$\hat{VBAR}_{RP} = \hat{P} \cdot \hat{VBAR}_P, \text{ where}$$

$$\hat{P} = -0.1943+0.9444 \cdot \frac{RH}{PH}+0.1010 \cdot \frac{1}{RH}+0.02757 \cdot \frac{RH}{SH}$$

In the above equations, TH is total height in feet, PH is pulpwood merchantable height in 100-in. sticks to an approximate 3.6-in. top diameter limit, SH is sawtimber merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit, RH is the residual number of pulpwood sticks above and beyond sawtimber sticks, and $\hat{P}$ is the predicted proportion of residual pulpwood VBAR. For trees with both sawtimber and residual pulpwood volume, we recommend that sawtimber volume be determined using $\hat{VBAR}_S = \hat{VBAR}_P - \hat{VBAR}_{RP}$. For trees with just sawtimber, Equation 3 above should be used. Pulpwood and residual pulpwood rough cord $\hat{VBAR}$s can be determined from the respective cubic foot $\hat{VBAR}$s using appropriate cu.ft./cd. conversion values.

The above equations can be used to develop tables as we have done in this paper or entered into a computer program to facilitate computer volume calculations for cruise data.

MICHIGAN
DEPARTMENT OF NATURAL RESOURCES
FOREST MANAGEMENT DIVISION

SUBJECT - VOLUME-BASAL AREA RATIO EQUATIONS

DATE - 30 September 87

TITLE - Volume-Basal Area Ratio Equations for Red Pine in Michigan.

AUTHORS - Gary W. Fowler, Professor of Biometrics, School of Natural Resources, University of Michigan, and Nemah G. Hussain, Timber Sales Specialist, Forest Management Division, Michigan Department of Natural Resources.

Background

The Michigan Department of Natural Resources (DNR) developed volume-basal area ratios (VBARs) expressed in volume units per sq. ft. in 1974 to be used in estimating pulpwood volume (cords) and sawtimber volume (board feet, International-¼ Rule) based on prism cruising. Three tally sheets were developed based on these VBARs (R 4137, 4144, and 4145). Because of perceived inaccuracies in the pulpwood VBARs and the time involved in using these tally sheets, the DNR is presently using Carlson's Formula for some pulpwood volume prism cruising. The formula for the Carlson Method when using a basal area factor of 10 is

$$\text{vol. in cds./ac.} = \frac{\text{Total no. of trees} + \text{Total no. of sticks}}{2}.$$

Purpose

The purpose of this paper is to present new total, pulpwood, sawtimber, and residual pulpwood cubic-foot VBAR equations and tables for red pine in Michigan. Rough cord VBAR equations and tables are also given for pulpwood and residual pulpwood.

Methods and Materials

Felled tree and/or standing tree measurements were made on a total of 3507 trees from 27 stands as follows:

1) 2341 trees from 16 stands in the Upper Peninsula (i.e., 5, 4, and 7 stands in the eastern, central, and western U.P., respectively), and

2) 1166 trees from 11 stands in the Lower Peninsula (i.e., 4, 4, and 3 stands from the northeast, northwest, and southern L.P., respectively).

Stands were selected from the above six regions to roughly represent the range of site index, age, stand density, average diameter at breast height (DBH), and average height found in Michigan. Measurements were made during May-August, 1983-1985.

For the 27 stands, site index varied from 45 to 75, age varied from 26 to 105 years, basal area/acre varied from 90 to 225 sq.ft., average DBH varied from 6.7 to 17.7 in., average total height varied from 33.2 to 86.0 ft., and average merchantable height to an approximate 3.6-in. minimum top diameter varied from 2.0 to 8.4 100-in. sticks.

For felled trees, DBH to the nearest 0.1 in., total height to the nearest ft., merchantable height to the nearest 100-in. stick to an approximate 3.6-in. minimum top diameter, and diameter inside (DIB) and outside (DOB) bark to the nearest 0.1 in. at the end of each stick were measured for each tree. For standing trees, measurements were taken at stump height (0.5 ft.), DBH height (4.5'), several upper stem taper breaks, approximate 3.6-in. DIB height, and the tree top using a Barr and Stroud dendrometer. A bark factor equation was developed using the felled tree data to estimate DIBs for standing trees (Fowler and Hussain 1987).

Merchantable height to an approximate 3.6-in. minimum top diameter is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum inside bark top diameter no smaller than 3.6 inches. This minimum top diameter was decreased for trees where the last stick had a minimum top diameter of 3.6 in. at a length of at least 6 ft. and a full 100-in. stick could be cut from the tree. For felled trees, the last stick sometimes had a minimum top diameter less than 3.6 inches. Merchantable height to an approximate 7.6-in.

minimum top diameter is defined as the number of 100-in. sticks that can be cut out of a tree with a minimum top diameter no smaller than 7.6 inches.

For each tree, cubic-foot volumes were calculated for each 100-in. stick using Smalian's formula. The volume of the butt stick was determined by breaking the stick into two pieces at DBH height, calculating the volume separately for each piece using Smalian's formula, and summing the two volumes. Pulpwood and sawtimber volumes were determined by summing up the volumes of sticks to approximate 3.6-in. and 7.6-in. top diameter limits, respectively. Total tree volume was determined by adding the volume of the tree top (assumed to be a cone) to the pulpwood volume. Residual pulpwood volume above and beyond sawtimber volume was determined as the difference between pulpwood and sawtimber volumes. The total pulpwood, sawtimber, and residual pulpwood VBARs were obtained for each tree by dividing the appropriate cubic-foot volume of the tree by the basal area in sq.ft. of the tree at 4.5 ft. from the ground. Pulpwood VBARs in rough cords were obtained by dividing cubic-foot VBARs by 77-84 cu.ft./cd., depending on the average DOB of all sticks for trees with merchantable heights varying from 1-10 sticks (Taras 1956, Avery and Burkhart 1983), and multiplying the result by 0.96 to compensate for the extra 4 in. of stick length beyond 8 ft. Residual pulpwood VBARs in rough cords were obtained by dividing cubic-foot VBARs by 79 cu.ft./cd. and multiplying by 0.96.

VBAR was regressed on various forms of tree height and DBH using multiple linear regression.

Results

The data set used to develop the regression equations consisted of approximately 80% of the trees in each of the 27 stands, yielding a total of 2789 trees.

The total VBAR equations were based on 2789 trees with DBH ranging from 3.6 to 23.9 in., total height ranging from 23.3 to 100.8 ft., and total cubic foot volume ranging from 1.08 to 105.05 cu.ft.

The pulpwood VBAR equations were based on 2774 trees with DBH ranging from 4.6 to 23.9 in., merchantable pulpwood height varying from 1-10 sticks, and pulpwood cubic-foot volume ranging from 0.65 to 103.68. A pulpwood tree had to have at least one 100-in. stick having a minimum top diameter $\geq$3.6 inches.

The sawtimber VBAR equations were based on 1742 trees with DBH ranging from 7.9 to 23.9 in., sawtimber merchantable height ranging from 1-9 sticks, and sawtimber cubic-foot volume ranging from 2.52 to 100.95 cu.ft. A sawtimber tree had to have at least one 100-in. stick with a minimum top diameter $\geq$7.6 inches.

The residual VBAR equations were based on 1671 trees (i.e., those sawtimber trees that had sawtimber volumes and pulpwood volumes). The number of residual pulpwood sticks varied from 1-6, 1-5, 1-4, 1-4, 1-4, 1-3, 1-3, and 1-2 for 1, 2, 3, 4, 5, 6, 7, and 8 sawtimber sticks, respectively. One tree with 9 sawtimber sticks had one residual pulpwood stick.

Total, pulpwood, and sawtimber VBAR prediction equations

A comparison of various multiple linear regression equations based on goodness-of-fit and simplicity indicated that the following prediction equations compared favorably to all other equations examined for total, pulpwood, and sawtimber VBARs:

1. Height independent variables

   $\hat{VBAR} = \hat{\beta}_o + \hat{\beta}_1 H + \hat{\beta}_2 \frac{1}{H}$, and

2. Height and DBH independent variables

   $\hat{VBAR} = \hat{\beta}_o + \hat{\beta}_1 H + \hat{\beta}_2 \frac{1}{H} + \hat{\beta}_3 D + \hat{\beta}_4 \frac{1}{D}$,

where $\hat{VBAR}$ is predicted VBAR, H is total height in ft. (TH), merchantable height in 100-in. sticks to an approximate 3.6-in. top diameter limit (PH), or merchantable height in 100-in. sticks to an approximate 7.6-in. top diameter limit (SH) for total, pulpwood, and sawtimber VBAR, respectively, and D is DBH in inches. $\hat{\beta}_o$ is the sample intercept or regression constant, and $\hat{\beta}_1$, $\hat{\beta}_2$, and $\hat{\beta}_3$ are the sample regression coefficients related to the various independent variables. There were no significant differences between the regions in the U.P. and regions in the L.P using the one-way analysis of variance for intercepts (β_o) and the two regression coefficients (β_1 and β_2) with the height only models for total, pulpwood, or sawtimber VBAR prediction equations using a level of significance α=0.05. Therefore, all of the data from the 27 stands were pooled for each type of volume equation to develop one pooled prediction equation for Michigan.

Table 1 shows the three cubic-foot VBAR prediction equations along with the sample sizes (n), standard errors of estimate ($s_{y\cdot x}$), and coefficients of

Table 1. Estimated intercepts ($\hat{\beta}_o$), regression coefficents $\hat{\beta}_1$ and $\hat{\beta}_2$, sample sizes (n), standard errors of the estimate ($s_{y\cdot x}$), and coefficients of multiple determination (R^2) for the total, pulpwood, and sawtimber cubic-foot VBAR prediction equations with independent variables based on height only.

Prediction Equation	$\hat{\beta}_o$	$\hat{\beta}_1$	$\hat{\beta}_2$	n	$s_{y\cdot x}$	R^2
(1) Total[a]	12.9975	0.3375	-285.1908	2789	2.66	0.834
(2) Pulpwood[b]	11.8783	3.2110	- 8.4061	2774	2.23	0.912
(3) Sawtimber[c]	11.4395	3.5298	- 7.1381	1742	1.85	0.953

[a] $\hat{VBAR}$ = 12.9975+0.3375·TH-285.1908·(1/TH).
[b] $\hat{VBAR}$ = 11.8783+3.2110·PH- 8.4061·(1/PH).
[c] $\hat{VBAR}$ = 11.4395+3.5298·SH- 7.1381·(1/SH).

determination (R^2) for height only models. Total, pulpwood, and sawtimber cubic-foot VBAR values for various heights are shown in Table 2. Pulpwood rough

Table 2. Total, pulpwood, and sawtimber VBARs in cu.ft./ sq.ft. for various values of total or merchantable height.

Total		Pulpwood		Sawtimber	
Height (feet)	VBAR	Height (sticks)	VBAR	Height (sticks)	VBAR
20	5.49	1	6.68	1	7.83
30	13.62	2	14.10	2	14.93
40	19.37	3	18.71	3	19.65
50	24.17	4	22.62	4	23.77
60	28.49	5	26.25	5	27.66
70	32.55	6	29.74	6	31.43
80	36.43	7	33.15	7	35.13
90	40.20	8	36.52	8	38.79
100	43.89	9	39.84	9	42.41
110	47.53	10	43.15	10	46.02

cord VBAR values are shown in Table 3. Table 3 also shows the pulpwood rough cord VBARs presently used by the DNR. The new VBARs are -3.5, 18.8, 12.9, 8.0,

Table 3. Rough cord VBARs for various numbers of pulpwood sticks from the new pulpwood VBAR prediction equation and from DNR tally sheets R 4137 and 4144.

Height (sticks)	VBAR		Height (sticks)	VBAR	
	New	DNR		New	DNR
1	0.083	0.086	6	0.348	0.333
2	0.171	0.144	7	0.383	0.368
3	0.227	0.201	8	0.417	0.391
4	0.271	0.251	9	0.455	--
5	0.311	0.296	10	0.493	--

5.1, 4.5, 4.1, and 6.6% larger than the VBARs in tables presently used by the DNR for trees with 1, 2, 3, 4, 5, 6, 7, and 8 sticks, respectively. These values were obtained by dividing the pulpwood VBAR values in Table 2 by 77, 79, 80, 81, 82, 83, and 84 cu.ft./cd. for trees with 1, 2-3, 4, 5, 6, 7, and 8-10 sticks, respectively, and multiplying the result by 0.96.

Table 4 shows the three cubic-foot VBAR prediction equations for the height and diameter models. Note that R^2 and $s_{y \cdot x}$ for these equations are larger and

Table 4. Estimated intercepts ($\hat{\beta}_o$), regression coefficents $\hat{\beta}_1$, $\hat{\beta}_2$, $\hat{\beta}_3$, and $\hat{\beta}_4$, sample sizes (n), standard errors of the estimate ($s_{y \cdot x}$), and coefficients of multiple determination (R^2) for the total, pulpwood, and sawtimber cubic-foot VBAR prediction equations with independent variables based on height and diameter.

Prediction Equation	$\hat{\beta}_o$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$	n	$s_{y \cdot x}$	R^2
(4) Total[a]	12.1891	0.4751	-96.0705	-0.7444	-28.9981	2789	2.27	0.879
(5) Pulpwood[b]	14.6394	3.9789	- 6.6553	-0.5816	-10.4024	2774	1.83	0.941
(6) Sawtimber[c]	19.9480	4.5690	-6.0728	-0.8525	-27.5071	1742	1.24	0.979

[a] $\hat{VBAR} = 12.1891+0.4751 \cdot TH-96.0705 \cdot (1/TH)-0.7444 \cdot D-28.9981 \cdot (1/D)$.

[b] $\hat{VBAR} = 14.6394+3.9789 \cdot PH- 6.6553 \cdot (1/PH)-0.5816 \cdot D-10.4024 \cdot (1/D)$.

[c] $\hat{VBAR} = 19.9480+4.5690 \cdot SH- 6.0728 \cdot (1/SH)-0.8525 \cdot D-27.5071 \cdot (1/D)$.

smaller, respectively, than for the respective equations based on height only (Table 1). Total, pulpwood, and sawtimber cubic-foot VBAR tables based on these equations are shown in Tables 5, 6, and 7, respectively, in the Appendix. A pulpwood rough cord VBAR table is given in Table 8 in the Appendix. The values in this table were obtained the same way Table 3 values were obtained from Table 2 values for the height only model for pulpwood VBAR. The same cubic-feet per rough cord values for merchantable heights of 1 to 10 sticks were used to obtain Table 8 as were used to obtain Table 3.

Residual pulpwood VBAR prediction equations

Residual pulpwood VBAR in a sawtimber tree was estimated using two methods: 1) Use of a prediction equation for residual VBAR, and 2) multiplication of the estimated proportion of residual pulpwood VBAR in a sawtimber tree by the estimated pulpwood volume in the tree.

Method 1 -- Two prediction equations were developed for residual pulpwood cubic-foot VBAR:

1. Height independent variables

(7) $$\hat{VBAR}=-10.8751+15.9228\cdot\frac{RH}{PH}+3.7199\cdot RH+6.4795\cdot\frac{1}{RH}$$

$R^2=0.953$, $s_{y\cdot x}=1.21$, n=1671

2. Height and diameter independent variables

(8) $$\hat{VBAR}=-31.5976+4.2443\cdot\frac{RH}{PH}+4.0902\cdot RH+5.7334\cdot\frac{1}{RH}$$
$$+0.6148\cdot D+201.5407\cdot\frac{1}{D}$$

$R^2=0.974$, $s_{y\cdot x}=0.90$, n=1671

where $\hat{VBAR}$ is predicted residual pulpwood VBAR and RH is the residual number of pulpwood sticks above and beyond sawtimber sticks.

Table 9 in the Appendix shows residual cubic-foot VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equation 7. Table 10 in the Appendix shows residual rough cord VBARs for various numbers of sawtimber and residual pulpwood sticks. VBARs from Table 9 were divided by 79 cu.ft./cd and then multiplied by 0.96 to obtain Table 10 values. A residual cubic-foot or rough cord VBAR table can also be developed from Equation 8 for various values of SH, RH, and D.

Method 2 -- Two prediction equations were developed for proportion of residual pulpwood VBAR in a sawtimber tree:

1. Height independent variables

(9) $$\hat{P}=-0.1943+0.9444\cdot\frac{RH}{PH}+0.1010\cdot\frac{1}{RH}+0.02757\cdot\frac{RH}{SH}$$

$R^2=0.990$, $s_{y\cdot x}=0.0215$, n=1671

2. Height and diameter independent variables

(10) $$\hat{P}=-0.2384+0.7804\cdot\frac{RH}{PH}+0.08480\cdot\frac{1}{RH}+0.03580\cdot\frac{RH}{SH}+1.2740\cdot\frac{1}{D}$$

$R^2=0.992$, $s_{y\cdot x}=0.0187$, n=1671

where $\hat{P}$ is the predicted proportion of residual pulpwood VBAR and SH is the number of sawtimber sticks. Table 11 in the Appendix gives values of $\hat{P}$ from Equation 9 for various numbers of sawtimber and residual pulpwood sticks.

Predicted residual pulpwood VBAR can be obtained by multiplying either Equation 9 or 10 times Equation 2 or 5, respectively. Tables 12 and 13 in the Appendix show cubic-foot and rough cord VBARs, respectively, for various values of SH and RH based on Equations 9 and 2. Table 12 values were divided by 79 and multiplied by 0.96 to obtain Table 13 values. Tables could also be developed based on Equations 10 and 5 for various values of SH, RH, and D.

For sawtimber trees with residual pulpwood volume, sawtimber cubic-foot VBAR can be obtained by subtracting predicted residual pulpwood VBAR (using Methods 1 or 2) from predicted pulpwood VBAR (Equations 2 or 5).

Validation

The data set used to validate the prediction equations consisted of the other approximately 20% of the trees in each of the 27 stands, yielding a total of 718 trees. For each volume equation, the average relative error as a percent ($\overline{RE}$) was calculated for each region and all regions pooled where

$$\overline{RE} = \sum_{i=1}^{n} RE_i \quad n$$

and $RE_i = [(\hat{VBAR}_i - VBAR_i)/VBAR_i]x100$, $\hat{VBAR}_i$ and VBAR are the predicted and actual VBARs for the i^{th} tree, and n is the number of trees for a region or all regions

pooled. The $\overline{RE}$ for all regions pooled along with the range for the 6 regions was determined for all VBAR equations.

For 718 trees, $\overline{RE}$ was 0.89% (range: -6.31 to 7.14%) and 0.42% (range: -4.09 to 3.09%) for the total cubic-foot VBAR equations based on height (Equation 1) and height and diameter (Equation 4) independent variables, respectively.

For 716 trees, $\overline{RE}$ was 0.81% (range: -3.84 to 4.40%) and 0.43% (range: -1.39 to 4.28%) for the pulpwood cubic-foot VBAR equations based on height (Equation 2) and height and diameter (Equation 5) independent variables, respectively.

For 443 trees, $\overline{RE}$ was 1.00% (range: -4.18 to 7.45%) and 0.05% (range: -0.94 to 2.84%) for the sawtimber cubic-foot VBAR equations based on height (Equation 3) and height and diameter (Equation 6) independent variables, respectively.

For 432 trees, $\overline{RE}$ was 4.98% (-7.13 to 21.20%) and 0.43% (-3.08 to 3.19%) for residual pulpwood cubic-foot VBAR equations based on height (Equation 7) and height and diameter (Equation 8) independent variables, respectively (Method 1). $\overline{RE}$ was 3.29% (-11.23 to 24.64%) and -1.85% (-9.08 to 5.23%) for residual pulpwood cubic-foot VBARs based on multiplying the proportion of residual pulpwood VBAR Equation 9 times the pulpwood VBAR Equation 2 (height independent variables) and on multiplying the proportion of residual pulpwood VBAR Equation 10 times the pulpwood VBAR Equation 5 (height and diameter independent variables), respectively (Method 2).

The relative errors for the equations with height and diameter independent variables were considerably smaller than the relative errors for the equations with just height independent variables.

The relative errors for the residual pulpwood VBAR models based on Method 1 were less variable over the six regions than the relative errors based on Method 2. However, the pooled average relative error was larger for Method 1 for the equation with only height independent variables and smaller for Method 1 for the equation with height and diameter independent variables.

One sample was taken in each of two red pine stands using a BAF-20 prism:

(1) Four points were selected from a red pine stand in Lower Michigan. DBH to the nearest 0.1 in. and merchantable height to the nearest 100-in. stick to an approximately 3.6" DIB minimum top diameter was measured for each "in" tree with DBH$\geq$4.6". DBH varied from 6.0-10.4 in., and merchantable height varied from 3-6 sticks. The average number of "in" trees per point was 8.5.

(2) Ten points were selected from a red pine stand in Upper Michigan. Merchantable height to the nearest 100-in. stick to an approximately 3.6" DIB minimum top diameter was measured for each "in" tree with DBH$\geq$4.6". Merchantable height varied from 5-7 sticks. The average number of "in" trees per point was 7.3.

Volume in rough cords was estimated for both stands using the new pulpwood prediction Equation 2 and VBARs currently used by the DNR (Table 3) with merchantable height as the independent variable, and Carlson's Formula. The new pulpwood prediction Equation 5 with DBH and merchantable height as independent variables was also used for the Lower Michigan stand.

Results for the two samples are shown in Table 14. For the Upper Peninsula stand, both our new equation and Carlson's Formula yielded estimates 4.6% higher than the DNR VBAR estimate. For the Lower Peninsula stand, our new Equations 2 and 5 and Carlson's Formula yielded estimates 8.6, 10.2, and 0.2% higher than the DNR VBAR estimate.

Table 14. Volume estimates in rough cords for the samples taken from 2 stands, one in the Upper Peninsula and one in the Lower Peninsula, based on DNR VBARs, our Equations 2 and 5, and Carlson's Formula.

Stand	DNR VBARs	Equation 2	Equation 5	Carlson's Formula
Upper Michigan	47.8	50.0	---	50.0
Lower Michigan	43.1	46.8	47.5	43.2

As expected, the new pulpwood VBAR formulas yielded estimates higher than the DNR pulpwood VBAR estimates. Our VBARs are larger than the DNR pulpwood VBARs for each height class except for the 1 stick class (Table 3). Differences for 2-4 sticks are larger than differences for 5-8 sticks.

Guidelines for Users

The new total, pulpwood, sawtimber and residual pulpwood VBAR prediction equations using DBH and height independent variables were more accurate than the respective equations using only height independent variables (Tables 1 and 4 and validation results). However, the average relative errors obtained with the height only VBAR equations are more than adequate for most situations.

Validation results indicated that the residual pulpwood VBAR equation (Method 1) yielded slightly better results than multiplying the proportion of residual pulpwood VBAR equation times the pulpwood VBAR equation (Method 2). Either method could be used to estimate residual pulpwood VBAR. However, an argument can be made for Method 2. The use of Method 2 to obtain residual pulpwood VBAR followed by determining sawtimber VBAR as the difference between pulpwood VBAR and residual pulpwood VBAR yields a compatible approach to total VBAR estimation. The use of the sawtimber VBAR equation and the residual pulpwood VBAR (Method 1) would not yield the total VBAR obtained by the pulpwood VBAR equation.

In the long run, our pulpwood rough cord VBARs would yield per acre estimates from 3.5% lower (all trees with 1 stick) to 18.8% higher (all trees with 2 sticks) than estimates based on the VBARs presently used by the DNR. In general, estimates from our VBARs would be about 8-10% higher for stands with the number of sticks varying from 1-4, and about 4-6% higher for stands with higher numbers of sticks.

Carlson's Formula is commonly used to estimate pulpwood rough cord volume per acre. Assume that all of the trees in a prism sample had the same merchantable height. Carlson's Formula would yield estimates 20.5, -12.3, -11.9, -7.3, -3.5, 0.6, 4.4, 7.9, 9.9, and 11.6% larger than estimates based on our new VBARs for 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 stick trees, respectively. Carlson's Formula (1) overestimates pulpwood volume for 1-stick trees and trees with 7 or more sticks and (2) underestimates pulpwood volume for 2- to 5-stick trees compared to our new VBARs. The difference between the 2 methods is small for 6-stick trees.

We recommend the use of the following cubic-foot VBAR prediction equations using height independent variables for most cruising situations:

1. Total VBAR - Equation 1 (Table 2).
2. Pulpwood VBAR - Equation 2 (Table 2, rough cord VBARs-Table 3).
3. Sawtimber VBAR - Equation 3 (Table 2).
4. Sawtimber and residual pulpwood VBARs - Method 2.
   - A. Residual pulpwood cubic-foot VBAR - Equation 9 x Equation 2 (Table 12).
   - B. Residual pulpwood rough cord VBAR - Convert cubic-foot VBARs to rough cord VBARs (Table 13).
   - C. Sawtimber cubic-foot VBAR - subtract the product of Equation 9 and Equation 2 from Equation 2 (subtract residual pulpwood value in Table 12 from pulpwood value in Table 2).

For those situations where extra accuracy is needed and the extra cost of measuring DBH is justified, we recommend use of the following cubic-foot VBAR equations using height and DBH independent variables:

1. Total VBAR - Equation 4 (Table 5).
2. Pulpwood VBAR - Equation 5 (Table 6, rough cord VBARs-Table 8).
3. Sawtimber VBAR - Equation 6 (Table 7).
4. Sawtimber and residual pulpwood VBAR - Method 2.
   A. Residual pulpwood cubic-foot VBAR (Equation 10 x Equation 5).
   B. Sawtimber cubic-foot VBAR (subtract the product of Equation 10 and Equation 5 from Equation 5).

Literature Cited

Avery, T. E. and H. E. Burkhart. 1983. Forest Measurements. McGraw-Hill Book Co., N.Y. p. 32-36.

Fowler, G. W. and N. G. Hussain. 1987. Bark factor equation for red pine. Michigan DNR Forestry Information Leaflet 1-87. 2 p.

Taras, M. A. 1956. Buying pulpwood by weight as compared with volume measure. U.S. Forest Serv. S.E. Forest Expt. Sta. Paper 74. 11 p.

Appendix

Table 5. VBAR table showing total cu.ft./sq.ft. for various combinations of DBH and total height.

DBH (inches)	Total Height in Feet								
	20	30	40	50	60	70	80	90	100
5	7.37	13.72	19.27	24.50	29.57	34.55			
6	7.59	13.94	19.49	24.72	29.79	34.77			
7	7.53	13.88	19.44	24.67	29.74	34.72	39.64		
8		13.65	19.21	24.44	29.51	34.49	39.42	44.30	
9		13.32	18.87	24.10	29.17	34.15	39.07	43.96	48.82
10		12.90	18.45	23.68	28.75	33.73	38.65	43.54	48.39
11			17.97	23.20	28.27	33.25	38.17	43.06	47.91
12			17.44	22.67	27.74	32.72	37.65	42.53	47.39
13			16.88	22.11	27.19	32.17	37.09	41.97	46.83
14			16.30	21.53	26.60	31.58	36.50	41.39	46.24
15			15.69	20.92	25.99	30.97	35.90	40.78	45.64
16			15.07	20.30	25.37	30.35	35.27	40.16	45.01
17			14.43	19.66	24.73	29.71	34.63	39.52	44.38
18			13.78	19.01	24.08	29.06	33.99	38.87	43.73
19			13.12	18.35	23.42	28.40	33.33	38.21	43.07
20			12.45	17.68	22.76	27.73	32.66	37.54	42.40
21			11.78	17.01	22.08	27.06	31.98	36.87	41.72
22			11.10	16.33	21.40	26.38	31.30	36.18	41.04
23			10.41	15.64	20.71	25.69	30.61	35.50	40.36
24			9.72	14.95	20.02	25.00	29.92	34.81	39.66
25			9.02	14.25	19.32	24.30	29.23	34.11	38.97
26			8.32	13.55	18.62	23.60	28.53	33.41	38.27
27			7.62	12.85	17.92	22.90	27.82	32.71	37.56
28			6.91	12.14	17.21	22.19	27.12	32.00	36.86
29			6.20	11.43	16.51	21.48	26.41	31.29	36.15
30			5.49	10.72	15.79	20.77	25.70	30.58	35.44

Table 6. VBAR table showing pulpwood cu.ft./sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 3.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
5	6.97	14.28	19.37						
6	6.74	14.05	19.13	23.67					
7	6.41	13.71	18.80	23.33	27.65				
8	6.01	13.32	18.40	22.94	27.25	31.45			
9	5.57	12.88	17.97	22.50	26.81	31.01	35.15		
10	5.11	12.41	17.50	22.03	26.35	30.55	36.68		
11		11.93	17.01	21.55	25.86	30.06	34.20	38.29	
12		11.42	16.51	21.04	25.36	29.56	33.69	37.79	
13		10.91	16.00	20.53	24.84	29.04	33.18	37.28	
14		10.38	15.47	20.01	24.32	28.52	32.65	36.75	
15			14.94	19.47	23.78	27.99	32.12	36.22	40.29
16			14.40	18.93	23.25	27.45	31.58	35.68	39.75
17			13.86	18.39	22.70	26.90	31.04	35.14	39.21
18			13.31	17.84	22.16	26.36	30.49	34.59	38.66
19			12.76	17.29	21.60	25.80	29.94	34.04	38.11
20			12.20	16.74	21.05	25.25	29.39	33.49	37.56
21				16.18	20.49	24.69	28.83	32.93	37.00
22				15.62	19.93	24.13	28.27	32.37	36.44
23				15.06	19.37	23.57	27.71	31.81	35.88
24				14.50	18.81	23.01	27.15	31.25	35.32
25				13.93	18.25	22.45	26.58	30.68	34.75
26					17.68	21.88	26.01	30.12	34.19
27					17.11	21.31	25.45	29.55	33.62
28					16.55	20.75	24.88	28.98	33.05
29					15.98	20.18	24.31	28.41	32.48
30					15.41	19.61	23.75	27.84	31.91

Table 7. VBAR table showing sawtimber cu.ft./sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 7.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
9	7.72	15.32	20.90	25.98	30.85				
10	7.17	14.77	20.36	25.43	30.30	35.07			
11	6.57	14.17	19.75	24.83	29.70	34.47			
12	5.92	13.53	19.11	24.18	29.06	33.83	38.54		
13	5.25	12.85	18.43	23.51	28.38	33.15	37.87	42.54	
14	4.54	12.15	17.73	22.81	27.68	32.45	37.16	41.84	
15	3.82	11.43	17.01	22.08	26.96	31.73	36.44	41.12	45.77
16		10.69	16.27	21.35	26.22	30.99	35.70	40.38	45.04
17		9.94	15.52	20.60	25.47	30.24	34.95	39.63	44.28
18		9.18	14.76	19.83	24.71	29.48	34.19	38.87	43.52
19		8.40	13.99	19.06	23.93	28.70	33.42	38.10	42.75
20		7.62	13.21	18.28	23.15	27.92	32.64	37.32	41.97
21			12.42	17.49	22.37	27.14	31.85	36.53	41.18
22			11.63	16.70	21.57	26.34	31.06	35.74	40.39
23			10.83	15.90	20.77	25.55	30.26	34.94	39.59
24			10.02	15.10	19.97	24.74	29.46	34.13	38.79
25			9.22	14.29	19.17	23.94	28.65	33.33	37.98
26				13.48	18.36	23.13	27.84	32.52	37.17
27				12.67	17.54	22.31	27.03	31.70	36.36
28				11.85	16.73	21.50	26.21	30.89	35.54
29				11.03	15.91	20.68	25.39	30.07	34.72
30				10.21	15.09	19.86	24.57	29.25	33.90

Table 8. VBAR table showing rough cords/sq.ft. for various combinations of DBH and merchantable height in sticks to an approximate 3.6" top diameter limit.

DBH (inches)	Merchantable Height in Sticks								
	1	2	3	4	5	6	7	8	9
5	0.087	0.174	0.235						
6	0.084	0.171	0.233	0.284					
7	0.080	0.167	0.228	0.280	0.328				
8	0.075	0.162	0.224	0.275	0.323	0.368			
9	0.069	0.157	0.218	0.270	0.318	0.363	0.407		
10	0.064	0.151	0.213	0.264	0.312	0.358	0.401		
11		0.145	0.207	0.259	0.306	0.352	0.396	0.438	
12		0.139	0.201	0.253	0.301	0.346	0.390	0.432	
13		0.133	0.194	0.246	0.294	0.340	0.384	0.426	
14		0.126	0.188	0.240	0.288	0.334	0.378	0.420	
15			0.182	0.234	0.282	0.328	0.372	0.414	0.460
16			0.175	0.227	0.276	0.321	0.365	0.408	0.454
17			0.168	0.221	0.269	0.315	0.359	0.402	0.448
18			0.162	0.214	0.263	0.309	0.353	0.395	0.442
19			0.155	0.208	0.256	0.302	0.346	0.389	0.436
20			0.148	0.201	0.249	0.296	0.340	0.383	0.429
21				0.194	0.243	0.289	0.333	0.376	0.423
22				0.187	0.236	0.283	0.327	0.370	0.416
23				0.181	0.230	0.276	0.321	0.364	0.410
24				0.174	0.223	0.269	0.314	0.357	0.404
25				0.167	0.216	0.263	0.307	0.351	0.397
26					0.210	0.256	0.301	0.344	0.391
27					0.203	0.250	0.294	0.338	0.384
28					0.196	0.243	0.288	0.331	0.378
29					0.189	0.236	0.281	0.325	0.371
30					0.183	0.230	0.275	0.318	0.365

Table 9. Residual pulpwood cubic-foot VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equation 7.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	7.29	10.42	14.39	18.36	22.29	26.17	30.02	33.85
2	4.63	7.77	12.00	16.24	20.39	24.47	28.47	32.43
3	3.31	6.17	10.41	14.72	18.97	23.14	27.24	31.27
4	2.51	5.11	9.27	13.59	17.87	22.08	26.22	30.31
5	1.98	4.35	8.42	12.70	16.98	21.21	25.38	29.49
6	1.60	3.79	7.75	11.99	16.26	20.49	24.66	28.79
7	1.31	3.34	7.22	11.41	15.65	19.87	24.05	28.19
8	1.09	2.99	6.79	10.93	15.14	19.35	23.52	27.66

Table 10. Residual rough cord VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equation 7.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.089	0.127	0.175	0.223	0.271	0.318	0.365	0.411
2	0.056	0.094	0.146	0.197	0.248	0.297	0.346	0.394
3	0.040	0.075	0.126	0.179	0.231	0.281	0.331	0.380
4	0.030	0.062	0.113	0.165	0.217	0.268	0.319	0.368
5	0.024	0.053	0.102	0.154	0.206	0.258	0.308	0.358
6	0.019	0.046	0.094	0.146	0.198	0.249	0.300	0.350
7	0.016	0.041	0.088	0.139	0.190	0.241	0.292	0.343
8	0.013	0.036	0.082	0.133	0.184	0.235	0.286	0.336

Table 11. Proportions of residual pulpwood cubic-foot VBAR in a sawtimber tree for various numbers of sawtimber and residual pulpwood sticks based on Equation 9.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.41	0.54	0.63	0.70	0.75	0.80	0.84	0.88
2	0.24	0.36	0.45	0.52	0.57	0.61	0.65	0.68
3	0.15	0.25	0.34	0.41	0.46	0.51	0.55	0.58
4	0.10	0.18	0.26	0.33	0.38	0.43	0.47	0.50
5	0.07	0.14	0.21	0.27	0.33	0.37	0.41	0.44
6	0.05	0.10	0.17	0.23	0.28	0.32	0.36	0.39
7	0.03	0.07	0.13	0.19	0.24	0.28	0.32	0.35
8	0.01	0.05	0.11	0.16	0.21	0.25	0.28	0.32

Table 12. Residual pulpwood cubic-foot VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equations 9 and 2.

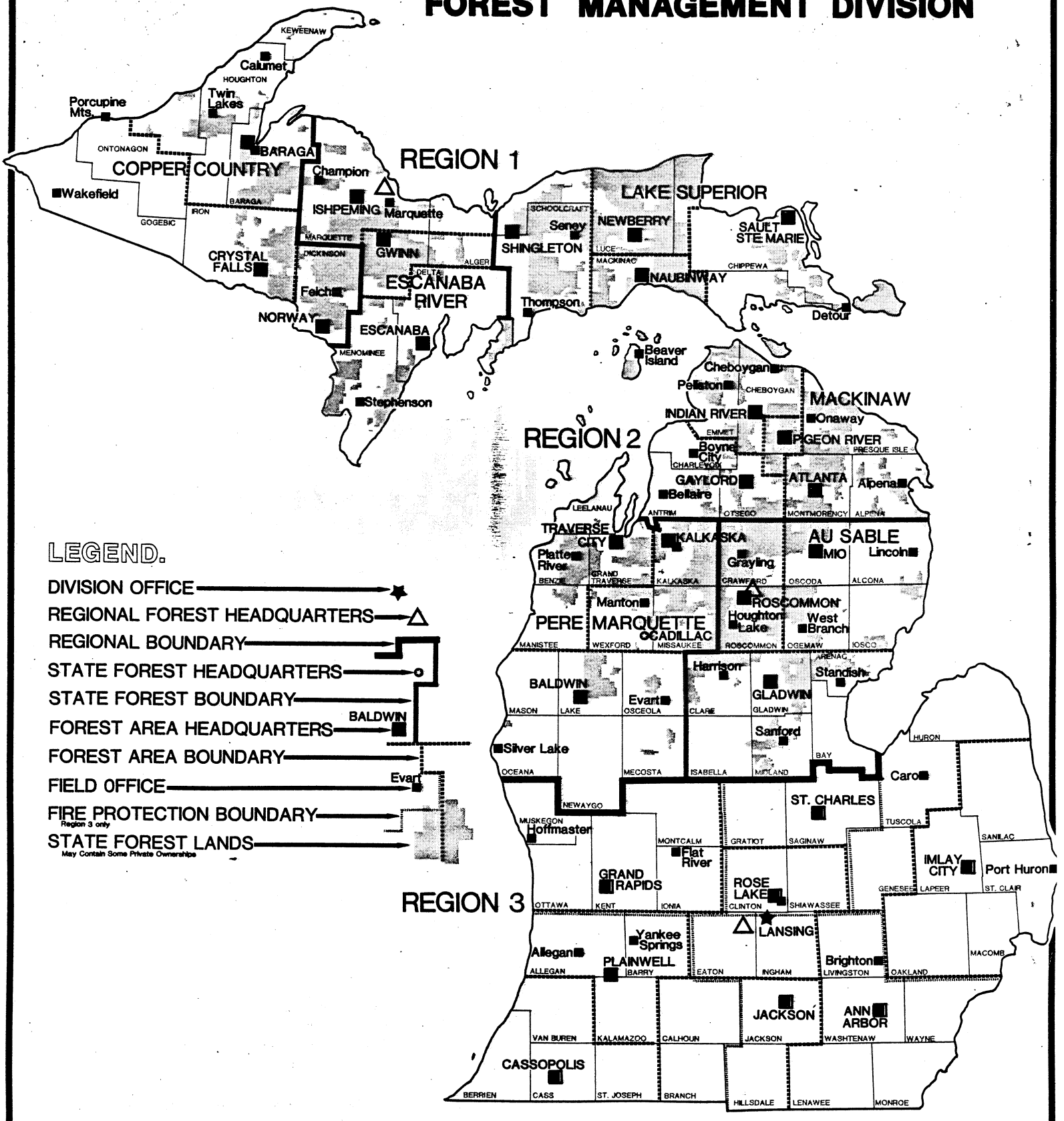
Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	5.73	10.12	14.26	18.29	22.33	26.44	30.65	34.99
2	4.40	8.05	11.74	15.34	18.88	22.40	25.94	29.52
3	3.44	6.62	10.09	13.50	16.87	20.21	23.54	26.87
4	2.69	5.49	8.78	12.07	15.34	18.57	21.79	25.00
5	2.06	4.54	7.67	10.86	14.05	17.21	20.36	23.50
6	1.53	3.70	6.69	9.80	12.91	16.02	19.11	22.19
7	1.04	2.94	5.80	8.83	11.88	14.94	17.99	21.03
8	0.60	2.24	4.98	7.93	10.93	13.94	16.95	19.95

Table 13. Residual rough cord VBARs for various numbers of sawtimber and residual pulpwood sticks based on Equations 9 and 2.

Number of Sawtimber Sticks	Number of Residual Pulpwood Sticks							
	1	2	3	4	5	6	7	8
1	0.070	0.123	0.173	0.222	0.271	0.321	0.372	0.425
2	0.053	0.098	0.143	0.186	0.229	0.272	0.315	0.359
3	0.042	0.080	0.123	0.164	0.205	0.246	0.286	0.327
4	0.033	0.067	0.107	0.147	0.186	0.226	0.265	0.304
5	0.025	0.055	0.093	0.132	0.171	0.209	0.247	0.286
6	0.019	0.045	0.081	0.119	0.157	0.195	0.232	0.307
7	0.013	0.036	0.070	0.107	0.144	0.182	0.219	0.255
8	0.007	0.027	0.060	0.096	0.133	0.170	0.206	0.242

MICHIGAN'S STATE FOREST SYSTEM

DEPARTMENT of NATURAL RESOURCES
FOREST MANAGEMENT DIVISION



LEGEND.

- DIVISION OFFICE
- REGIONAL FOREST HEADQUARTERS
- REGIONAL BOUNDARY
- STATE FOREST HEADQUARTERS
- STATE FOREST BOUNDARY
- FOREST AREA HEADQUARTERS (BALDWIN)
- FOREST AREA BOUNDARY
- FIELD OFFICE (Evart)
- FIRE PROTECTION BOUNDARY (Region 3 only)
- STATE FOREST LANDS (May Contain Some Private Ownership)