SAMPLING PROBLEMS IN THE MICHIGAN CREEL CENSUS

HOWARD D. TAIT

1953

ABSTRACT

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The objectives of this study were the critical review of sampling methods used to obtain catch records in the Michigan Intensive Creel Census and the determination of procedures for measuring the precision of estimates of the catch of fish per unit of effort, the total fishing intensity, and the total catch. Accurate and efficient sampling methods are needed because the Intensive Creel Censuses conducted on test waters provide a basis for fish management policies which affect many lakes and streams in the state.

The results of intensive creel studies of two lakes in southern Michigan are presented in this study. Included are records covering all angler activity (2214 anglers) on Bear Lake in 1941 and partial census records obtained through a sampling program in the years 1947 to 1952. Bear Lake is a small lake in Hillsdale County. Similar records are presented for comparison from a creel census of Pontiac Lake, which is a large, heavily fished lake in Oakland County (an estimated 40,000 anglers fished on this lake during the summer season of 1948).

These records were examined for information about the variability of catch-per-hour data, and of angler- and boat-count data, to determine the sample sizes required and to construct efficient schedules for conducting other censuses on a sampling basis. Sources of bias and factors affecting experimental design are identified and discussed. The efficiency of estimating the average catch per hour per angler and the average number of boats per hour by random sampling is compared

with the efficiency obtained by stratified random sampling with optimum allocation. The effects of sampling from non-normal distributions are also discussed. Methods for combining data and for computing confidence limits for estimates of total angler hours and total catch of fish are reviewed, and the precision actually attained for these quantities in the census of a representative lake is presented.

Several conclusions and recommendations made in this study are:

(1) The method of stratified random sampling, with allocation of the census effort among days according to the amount of fishing and the degree of variability of catch-per-hour and boat-count data, is more efficient than the method in which equal census effort is given to each type of day.

(2) The number of angler contacts made in a schedule that includes three half-days each week is usually adequate for estimating the mean catch per hour in the summer season but inadequate in other seasons.

(3) The poorest estimates in terms of confidence limits of the mean were obtained for boat counts, and it is probably in this area that the greatest improvements in the intensive census can be made.

(4) The non-normality of distributions of catch data may be ignored in comparisons of means because sample sizes are large but should be considered when estimates of variance and statements of the probability of events are made.

(5) Methods for computing estimates of the average catch per hour per angler should include provision for weighting samples according to fishing intensity.

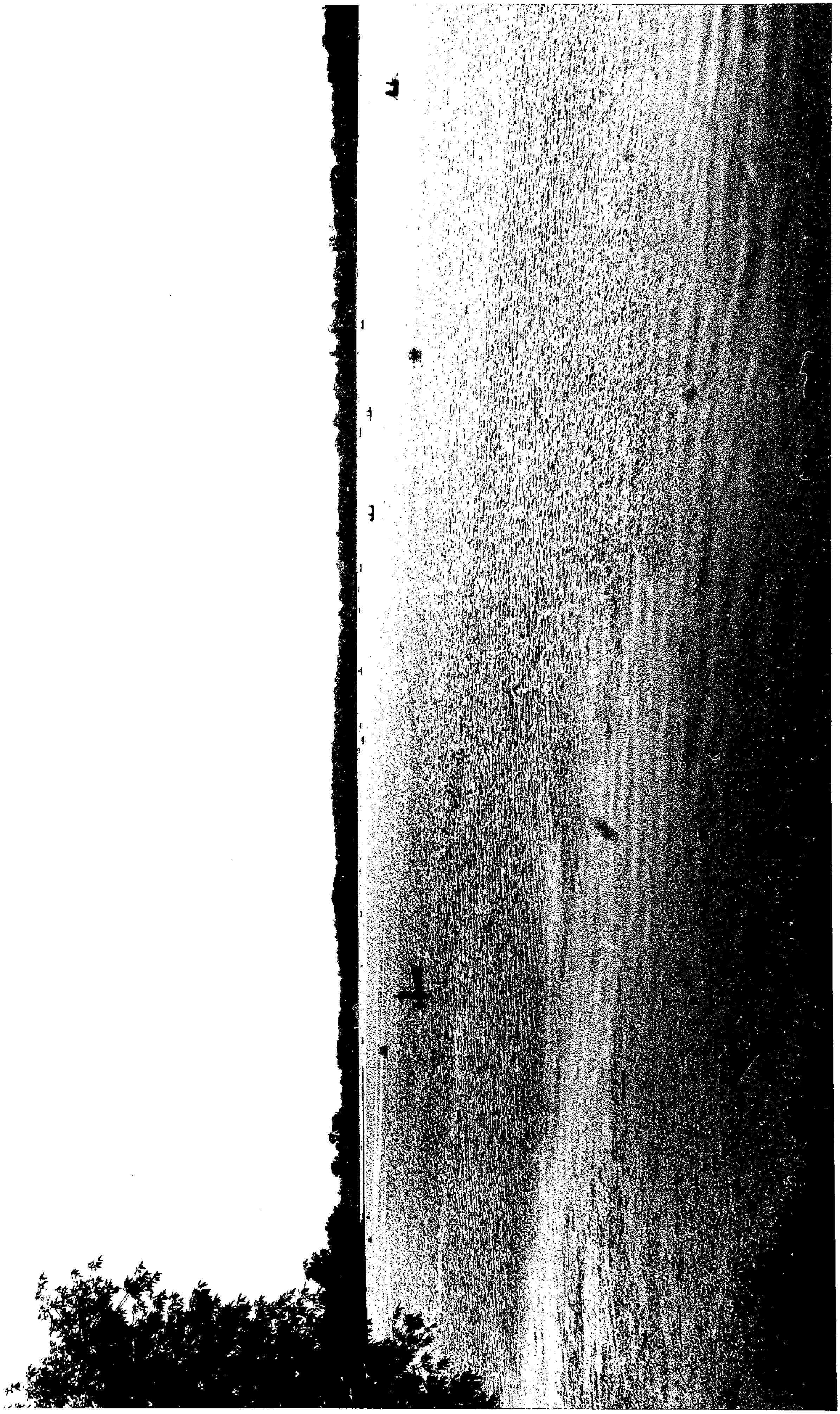
Report No. 1394

Sampling Problems in the Michigan Creel Census

By

Howard D. Tait

Frontispiece. Opening Day of Bass Fishing Season on Whitmore Lake, June 20, 1953



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A dissertation submitted in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy in the
University of Michigan

1953

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ACKNOWLEDGMENTS

The research described herein was accomplished while the author was employed as a graduate fellow by the Institute for Fisheries Research of the Michigan Department of Conservation. Generous financial aid and office and laboratory space were provided by the Institute. Grateful acknowledgment is extended to Dr. A. S. Hazzard, Director, for the unusual opportunity provided to conduct this study, and to Drs. Gerald P. Cooper and Frank F. Hooper for critical advice on many problems. Dr. Karl F. Lagler, Chairman of the Department of Fisheries, School of Natural Resources, gave many helpful suggestions and generously provided the use of his personal library. The creel census records used in this study were from Institute files. Computing machines used in certain analyses were made available at the Statistical Research Laboratory by Dr. Cecil C. Craig. Mr. William Cristanelli redrafted the figures and printed the photographs. Acknowledgment is also due other members of the doctoral committee, University faculty, and Institute staff for suggestions and criticism.

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INTRODUCTION

Importance of Yield Studies

Yield studies have become an important tool of fishery research and management. In fact, most of the problems which confront fisheries workers are concerned with the measurement and interpretation of fishing quality. If fishing is poorer than expected, that is, if the quantity of fish removed and the catch per unit of effort are low, one must examine the fish stock, the fish environment, and angler activity in an effort to determine the cause. Usually this is accomplished, in part, through one of the various forms of creel census. Catch statistics and biological data gathered in a creel census reveal much about the size and composition of the fish population in a lake and the portion of the population removed by anglers. The returns to anglers are usually measured in terms of the catch of fish per unit of fishing effort, the total catch of fish and the number of anglers sharing that catch, and the total amount of time spent fishing. If creel studies are continued over a period long enough to identify fluctuations of fish populations and variations in the catch, the expected yield to anglers can be established. The success or failure of management activities may then be judged on the basis of returns to anglers.

Although creel censuses have had wide application to fishery problems the need still exists for the development of accurate and practical means of estimating the yield by modern sampling methods.

The Problem

The Michigan Department of Conservation, through its Institute for Fisheries Research, measures the fishing quality of certain lakes and streams by means of an intensive creel census. The purpose of this program is to identify changes in fishing success that may result from new fishing regulations and other management devices. The criteria used to measure fishing quality are: the average catch per hour, the proportion of successful anglers, the species composition of the catch, and the total number of fish caught. Additional data on age and growth, on spawning success, and on the number of anglers are gathered to help in the evaluation. The test lakes under census were selected as being representative of Michigan waters. The results of investigations on these lakes influence fish management policies in the state. Thus, it is essential that accurate information be available from these lakes. At present the yield is estimated from samples rather than from a complete creel census.

In this paper the sampling method used in the Michigan Intensive Creel Census will be discussed, its sources of bias identified, and an appraisal made of the precision of estimation obtained. Specifically, an analysis will be made of the method for sampling to determine the average catch per hour, the total fishing intensity as measured by boat counts, and the total catch of fish. Certain problems associated with each of these estimates will be considered, and methods for computing confidence limits will be presented. Several practical sampling schedules for collecting catch records will be evaluated on the basis of the precision actually obtained.

The Development of Creel Census Methods

The development and application of creel census methods are relatively recent and have taken place for the most part since 1930. Early interest in the subject developed in response to questions about the suitability of waters in which hatchery fish were being planted and the desirability of fish stocking. The importance of angler catch records as a measure of the success or failure of fish planting operations was recognized as early as 1927 in Michigan. In that year, the Department of Conservation initiated a system of collecting records to determine the quality of fishing. By 1934 there was a growing awareness elsewhere of the need for a measure of the angler's catch (e.g., Clark, 1934). At this time fishery workers in California, Michigan, Minnesota, New York, and Vermont were also attempting to learn more about the supplies of fish in their waters from anglers. An early intensive creel study of an individual body of water was carried out in Vermont on the Middlebury River in 1933 (Lord, 1946). Eschmeyer (1935) studied the fish catch in a Michigan lake by means of a complete creel census conducted by U. S. Civilian Conservation Corps ("C.C.C.") personnel. He defined many precepts of creel census as an aid to fish management. Needham (1937) also emphasized the value of creel data and advanced ideas concerning census methods which are still appropriate. He stated, "The measurement of the catch per unit of effort affords a definite means of determining the trend of the fishing in an intensively fished lake and can be based on an annual random sample of the catch." Needham (<u>ibid.</u>) stressed that both successful and unsuccessful anglers should be recorded and that enough records should be collected to be statistically representative. He recognized the importance of collecting data year after year, and also compared partial and complete creel censuses.

Once the value of catch records was recognized and generally accepted as an appraisal device, creel census projects were organized in many states and applied to many types of problems. By this means, for example, Lord (1935) was able to demonstrate over-intensity of fishing on a Vermont test trout stream. Hazzard and Eschmeyer (1937), using a census method, concluded that winter fishing was not generally harmful to summer fishing in Michigan. In 1937 the results of other creel censuses conducted in California, Wisconsin, and New Hampshire were reported to the American Fisheries Society. Hazzard and Shetter (1939) studied the results of planting legal-size trout in the Pine River in Michigan by an intensive system which covered a large proportion of all fishing on the river during the study period. Also, in that year, Frey, et al., reported the results of a summer creel study of take and angling pressure of two large Wisconsin lakes. Additional investigators have used catch data to study the effect of moonlight and/or climatic conditions on fishing success: e.g., Mottley (1938), Wright (1945), and Cooper (1953).

Several methods of conducting creel censuses evolved as various investigators attempted to measure the yield from fisheries in different areas and under different field conditions. Each was characterized both by positive values and obvious limitations and all are in current use. The steps in the development of these methods seem to have been in order as follows:

(1) VOLUNTARY. Initial efforts to learn something about the kind and quantity of fish caught by asking fishermen to report voluntarily.

(2) COMPULSORY. Adoption of a compulsory system requiring that anglers report their catch.

(3) EXTENSIVE. In the extensive or general census, fishermen are contacted from most types of waters over a large area. Few records are obtained for a given body of water, but collectively over a large area many returns are gathered.

(4) INTENSIVE. The need for exact information prompted adoption of more detailed studies on certain waters. In intensive programs, complete enumeration of all fishing activity is attempted.

(5) SAMPLING. Increasing demand upon available funds and personnel for census studies led to the development of sampling methods for gathering catch information.

A discussion of this sequence and the investigators chiefly responsible follows.

The earliest attempts to secure catch data depended upon angler cooperation in a voluntary reporting system. It soon became apparent that a voluntary system would not always yield enough records for an intensive study. Lord (1946) described his early unsuccessful attempts to get angler cooperation and compared voluntary census returns with records obtainable by use of a combination fishing permit and compulsory statement on effort and success. In contrast, California early employed a voluntary system of creel census, Croker (1937). Curtis (1940) compared this voluntary system with mandatory ones, and discussed the reliability of the estimates obtained, and the disadvantages and uncertainties of the procedure.

The compulsory permit system which grew out of the experiences with voluntary angler reports has been eminently satisfactory and continues to be a method of getting perhaps the most complete and reliable records which can be obtained. This is particularly true when the method is used in conjunction with a strategically located and adequately staffed checking station. It is costly, however, and is perhaps undesirably restrictive upon angler activities.

The method of General Creel Census in securing catch records appears to have originated in Michigan in 1927 and has been continued to date. With some modification, it has been used frequently in other states and in provinces of Canada. In this procedure, conservation officers (and at times, other workers) collect catch records from anglers in the field from all parts of the state. Although data gathered in this manner yields useful information about the geographic distribution of fish, and indicates apparent trends of fishing quality, they have rather serious limitations. The returns are variable and the low number of records collected from any one water area allows only crude estimates of fishing quality, and usually gives no information about the total yield.

It may be said that the methods of intensive creel census were developed to overcome the foregoing limitations. As previously indicated, they are of two major types: complete censuses covering all angler activity, and partial ones based on sampling programs attempting to secure records representative of all fishermen. The analyses of the fish catch by Eschmeyer (1935) were based on records obtained by a nearly complete intensive procedure. C.C.C. enrollees, under supervision, contacted nearly all anglers fishing Fife Lake in Michigan. All anglers were observed and the small portion not actually contacted was noted. Thus, the total yield of fish and the efficiency of anglers were both quite accurately determined.

Any widespread application of the complete, intensive method is extremely costly, probably prohibitively so. As a result, various sampling programs were employed, but, for the most part, these were not based on sound statistical procedures. It is probable that the results of some investigations were unreliable and that the conclusions which were made were unwarranted because of this inadequacy. However, investigators were usually aware of the need for improved sampling methods. King (1939: p. 94) stated, "The complete census is being conducted on only three streams this season, but we are attempting to obtain representative samples of fishing returns on other streams whenever possible." Hazzard and Shetter (1939) described census records collected on the Pine River as not complete, but believed them to be an adequate statistical sample of the season's fishing. Eschmeyer (1939) cited personnel capabilities as a factor which limited the comparative reliability of data for the several years. Similar comments gleaned from fishery literature typify the loose status of creel census sampling programs during the period 1935 to 1941. "It is estimated that the catch records are 95 percent complete." "The data obtained seem reliable with reference to distribution and size of sample" "....Both the creel census and the fishermen count are only approximately correct....," etc. Thus, some early investigators were aware of the possible applications of sampling methods to creel censuses. Eventually, attempts to develop more precise statistical treatment of census data followed.

Among the first investigators to apply statistical methods to creel census data was Mottley (1938) who used the statistic "t" to test differences in fishing quality during phases of the moon. Curtis (1940) considered the problem of non-response from anglers. The work of Schuck (1942) stands out because of his use of transformations to normalize data and his adaptation of experimental designs, analysis of covariance, and the F-test of significance to yield studies. Wright (1945), working on the effect of moonlight on fishing success, showed the applicability of certain statistical methods to creel studies. He paired catch-per-hour data from a partial census and studied the mean difference with its standard error, and used a "t" test of significance. Earlier Wright (1943) noted that in some situations catch-per-hour data can lead to erroneous conclusions unless statistically appraised. Tarzwell and Miller (1943) studied the amount and variation of fishing in the lower T.V.A. reservoirs by a method of "stratified" random sampling. Sample counts were made, from a moving motorboat, of bank and boat fishermen, and total fishing intensity was estimated. The authors recognized the great variability in fishing intensity, in that standard errors of mean counts varied from 8 to 100 percent of the mean. They described (1943: p. 248) the sampling method as follows: "In order to distribute equally all factors responsible for the variation in fishing intensity, the period over which counts were made was 'stratified' by dividing it into 10-day periods during which time a complete count of the three reservoirs was made." By stratification these workers meant that for each 10-day unit the section to be counted and the day it was to be counted were determined by drawing numbers. This method enabled them to make three complete counts on each reservoir per month. This use of the term "stratified" is not in accord with the accepted statistical meaning of the word, but their intent is clear. In general their work is an important contribution to the development of methodology in this field. However, their statements concerning standard error of the mean count are vague, and more importantly the tabular data presented on the calculated number of fishing trips seem to ignore the large standard errors.

Some of the recent workers (e.g.; Calhoun; 1950, Moore, Cope and Beckwith, 1952) concerned with yield studies have taken full advantage of modern statistical methods, being aware of the inherent variability of catch data and errors involved in estimating the yield. However, many contemporary studies are still based on catch data which at best are of limited application. Some workers apparently resist statistics as a useful tool, and gather catch records without recourse to sampling theory or methods. The results of such investigations and the conclusions drawn from them would elicit more confidence if statements about the adequacy of the sampling program and the precision of the estimates obtained were included in the published account.

Sampling Applied to the Intensive Creel Census

Method of Obtaining Catch Records

Several types of creel studies are conducted on Michigan's lakes and streams, including the General, Special Trout Stream, and Intensive Lake Censuses. The Intensive Creel Census was designed to provide facts about fish and fishing from the "test" lakes previously described. A considerable portion of all fishing activity on these lakes is included in the census, and estimates of the average catch per hour, the total number of angler hours, and the total yield of fish, are possible. On Michigan's test lakes, a census clerk contacts anglers and obtains information on the number of hours spent fishing, the number of each species of fish caught, the method of fishing, the kind of bait used, and so forth. Each angler-contact provides information about the catch per hour. The total number of angler hours spent on the lake is determined indirectly through boat counts made at frequent intervals. Approximations of the total catch are derived from the previously obtained estimates of average catch per hour and total angler hour. The sampling method used and the portion of anglers interviewed has varied somewhat during the several years that the census has been in operation (see Christensen, 1953). At present the days on which the census is to be conducted are selected systematically (usually starting with the opening day of the fishing season), skipping uniform gaps of days. Each day of the week appears in the sample about equally during the season (Table 1). Allowance is made for days off (i.e., vacation days for the technician) and at times the schedules are changed slightly because of unforeseen events. On the days selected for census, the technician contacts anglers at random as they complete their fishing trips and makes periodic boat counts.

It is assumed that the days that are censused are representative of all days. That is, the sample means obtained on days when anglers are contacted are unbiased estimates of the true averages for all days. Similarly, it is hoped that the method of selecting anglers as they leave the lake will result in unbiased average values for the catch per hour and the number of anglers per boat.

Variability of Catch Data

The precision and accuracy of seasonal catch records would not be questioned if it were practical to obtain complete coverage of all angler activity on a lake. The inherent variability of yield data would still remain to influence fiducial statements about the averages, but if the objective is to learn how many fish are taken by anglers, and at what rate they are removed during a given period, then a complete creel census will yield exact information. However, if it is

Table 1. Systematic Method of Selecting Days and Hours on Which to Conduct a Creel Census

Day of the Week	Hours of the Day															
	A. M.								P. M.							
	7	8	9	10	11	12	1	2	3	4	5	6	7	8	9	10
Sunday	X	X	X	X	X	X	X	X								
Monday																
Tuesday									X	X	X	X	X	X	X	X
Wednesday																
Thursday	X	X	X	X	X	X	X	X								
Friday																
Saturday									X	X	X	X	X	X	X	X
Sunday																
Monday	X	X	X	X	X	X	X	X								
Tuesday																
Wednesday									X	X	X	X	X	X	X	X
Thursday																
Friday	X	X	X	X	X	X	X	X								
Saturday																
Sunday									X	X	X	X	X	X	X	X
Monday																
Tuesday	X	X	X	X	X	X	X	X								
Wednesday																
Thursday									X	X	X	X	X	X	X	X
Friday																
Saturday	X	X	X	X	X	X	X	X								

considered in proper perspective as only one out of all possible creel censuses of lakes under similar conditions, the complete census itself becomes a sample. Hence, it is necessary to study catch records under different conditions of fishing pressure and over several years, in order to appraise the nature of their variability. It is impossible, however, to measure the catch under all conditions of fishing pressure because anglers may or may not be present to fish the study area. Experiments can be continued over a period of years to measure the variation between years, but the conclusions drawn still contain the element of probability.

Decision to Sample

If the fishing quality that a lake produces during an interval can be estimated with chosen precision from a census of only part of the fishing activity, it is obviously more efficient to conduct the specified sampling procedure than to operate a complete creel census. Further, if the amount of effort required to determine the average catch and the total yield can be reduced, available personnel may cover more lakes or perform other duties. The decision to sample is not easy to make on the basis of efficiency alone. Perhaps only on large, heavily fished lakes is the decision obvious. There, it is often impractical to interview more than a small proportion of the anglers because of the physical nature of the lake and the amount of fishing activity. If anglers enter and leave a fishing area at many points, or if there is an extensive development of cottages along the shore, the difficulty of contacting all anglers is extreme. These are obvious considerations in a decision to sample. The relationships between the variability of catch data, the desired precision of estimates, and the required sample size, are more obscure but also cause concern. Further, it will be shown that a sample of the size needed to achieve desired accuracy may be impossible to obtain.

Another consideration is the comparative expense of the two methods. The cost of a sample census may be greater than that of a complete count. The initial planning, execution, and analysis of sample censuses require treatment by specially trained personnel. In some situations these limitations, combined with the requirement of large samples, may indicate that a complete census is most practicable.

Factors Governing Experimental Design

Three indices of fishing quality and intensity--total catch, total effort, and catch per unit of effort--are of primary importance in the Michigan creel census. The number and kind of samples necessary to estimate these indices largely determine the census schedule. Certain other data relating to the species composition of the catch, the distribution of angling effort and the type and success of fishing gear used, are collected, but the sampling effort required for the primary estimates will ordinarily be adequate for the less important ones. The objective is a sampling program which will produce unbiased estimates of all indices with usable precision and minimum cost.

If nothing is known about the population, samples of the anglers and of the fishing effort should be selected by unrestricted random sampling without replacement, but a pattern of stratified random sampling can be used if separate strata can be identified. Stratified random sampling results in practical census schedules for field personnel, and may yield a smaller variance of means and consequently improved confidence limits for estimates. This is true because the number of anglers varies between and within days (Figures 1 and 2).

Figure 1. Distribution of Anglers by Hours During the Summer Season, Bear Lake, 1941

Figure 2. Distribution of Anglers by Days During the Summer Season, Bear Lake, 1941

Figure 3. Distribution of Anglers by Months, Bear Lake, 1941

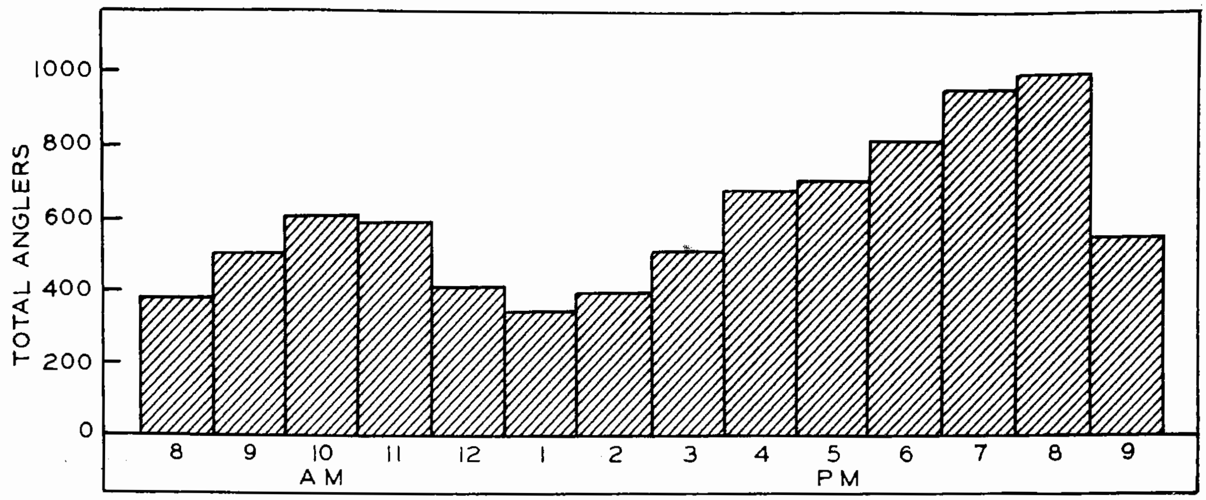


Figure 1.

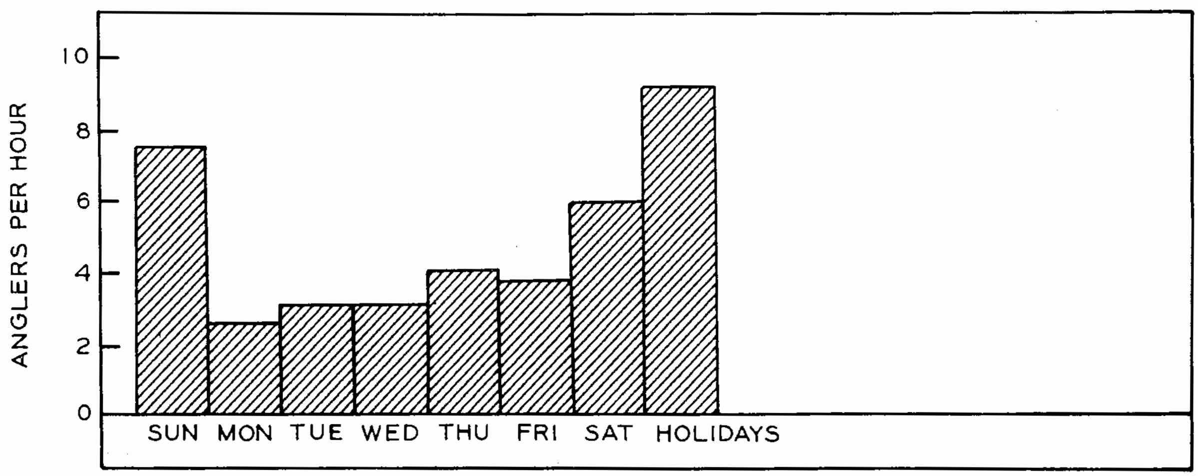


Figure 2.

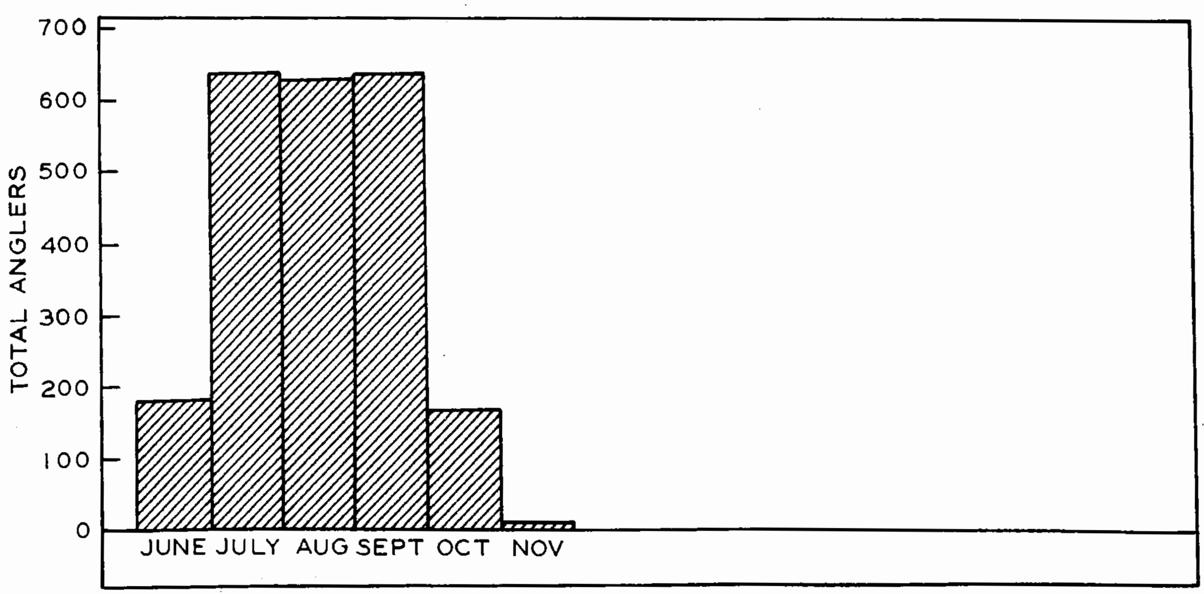


Figure 3.

Gains in efficiency may result from stratifying the population of anglers by hours or by days.

The size of sample necessary, and therefore the number of hours and days to include in the census schedule, will depend on the variability of the catch data, the precision of estimating desired, and the confidence level at which statements about the precision will be accepted. For example, an estimate of the average catch per hour may be desired within plus or minus one-tenth of a fish per hour. That is, enough anglers are to be contacted to insure that the probability is 0.95 that the true average is within 0.1 fish per hour of the sample mean. This can be a rather exacting requirement and such precision is often difficult to obtain in seasonal estimates. The difficulty stems from the extreme variability of catch-per-hour data. Even if all but a few anglers are included in the sample, the uncertainty of the catch of the excluded anglers prohibits very positive statements about the true average. Either the 95 percent probability level, or the confidence interval of 0.1 fish per hour will often have to be relaxed if adequate samples are unobtainable. At this precision, the variability of catch-per-hour data may require contacts with more anglers than are present during certain periods.

<u>Sources of Bias</u>

It is necessary to guard against bias in the selection of anglers to include in samples. A sample is biased if certain anglers or groups of anglers have a consistently better opportunity to be included than do other anglers. The elimination of bias is important because the error introduced is systematic and may be considerable. Its magnitude is usually unknown and unlike sampling error it cannot be predicted with confidence limits. Two sources of bias will be discussed: the census takers in the field, and the method of choosing the sample.

In theory, the anglers to be contacted for catch information are choosen at random from the population of anglers on the lake during the study period. In practice however, randomness may be difficult to attain. It is often easier to contact some anglers than others. For example, it is convenient to interview anglers at points of public access to the lake, but these anglers may not be typical of all anglers (Figure 4). They may be better or poorer fishermen on the average than cottage owners or anglers fishing from boat liveries, and should appear in the sample only in the proportion that they exist in the total population of anglers. Therefore, one must be cautious about securing samples at these locations to the exclusion of others.

Intensive creel censuses are usually continued over several years to learn something of the variability and changes in fishing that occur with time. As these studies are continued cottage owners and other local fishermen become familiar with the objectives of the census and the activities of the census clerks. Most of them are then quite cooperative in giving the desired information, some even to the point of seeking out the census clerk so that he may record their catch. Although such cooperation indicates public acceptance and interest in the project, there is the possibility that bias can be introduced into samples under such conditions. The cooperative anglers may appear in the samples more often than would be expected if the selection of anglers was entirely at random. Bias would occur if the success of these cooperative anglers differs from that of other anglers.

The manner of recording the amount of time an angler spends fishing may be considered a source of bias. Fishing time can include

Figure 4. View of Public Fishing Site at Whitmore Lake

(Photo by G. P. Cooper)

Figure 4

the time actually spent fishing, i.e., presenting the bait to the fish, or it may include time spent assembling tackle and traveling to the angling site. The time to be recorded should be decided in advance of the census and will depend on the type of study and to some extent on prior knowledge of the fishery. Usually all of the time spent on the water is included as fishing time or angler hours. Even though there may be an inefficient fishing period during which anglers are assembling tackle and traveling to their favorite fishing spots no adjustment for this time seems necessary. Angler behavior appears to be about the same on similar lakes from year to year, and any error introduced in catch-per-hour data by including all angler time on the water is probably consistent in data being compared. It is important, however, that the procedure be standardized for all study areas so data will be comparable.

The precision of measuring fishing time is seriously open to question in any system which depends on the anglers' memory. Fishermen ordinarily do not know, upon completing a fishing trip, the exact time at which they started to fish.

The commonest bias in catch records results from failure to include unsuccessful anglers in proper proportion. Failure to do so will produce inflated catch estimates. This type of error occurs chiefly in voluntary angler returns. It is not likely to be as important in the intensive census conducted by trained personnel, but certainly is possible. It is a commonly observed fact in sampling methods that more reliable results are obtained if the selection of individuals to appear in samples is mechanical (tables of random numbers, etc.) rather than left to the discretion of the observer.

Bias may occur when the schedule for conducting the census, i.e., the sample design, allows certain days or hours to appear consistently more often than others in the sample. Fishing quality and the intensity of fishing varies on a lake among certain periods of the day, and also between days. The selection of days and hours on which to obtain catch records, and the methods of combining data need careful consideration if unbiased estimates of yield are to be obtained. Problems of this type will be taken up in the following sections.

An estimate of the average catch per hour per angler is not only an index to fishing quality but is very important in the calculation of estimates of the total catch of fish and total number of anglers. Therefore, the problems of sampling and sources of bias in obtaining this value will be considered first.

PROBLEMS OF OBTAINING CERTAIN ESTIMATES FROM SAMPLES IN THE INTENSIVE CREEL CENSUS

Method and Problems of Estimating Average Catch per Hour per Angler

Method of Deriving Estimate from Field Data

There are two methods for computing the average rate of catch. (1) In one method the number of fish caught divided by the number of hours spent fishing is computed for each angler and then all such records are summed for the season or other study period. This simple arithmetic average of all individual catch per hour values is defined as the average catch per hour per angler.

(2) In the other method the total sum of fish caught by all anglers is divided by the total number of hours fished by all anglers. This quotient is called the average catch per hour. Of these two expressions for rate of catch, catch per hour per angler is more useful

because the variability of the data can be measured for use in statistical analyses. For this reason it is used in the Intensive Creel Census and in this paper. The method of computing the average catch per hour per angler is not complex, but as will be shown in a subsequent section the method of collecting catch records for the computation may affect its accuracy.

Examination of Assumptions and Problems of Method

A basic assumption of the sampling program is that the anglers who are contacted for a sample of the catch per hour per angler are randomly selected from all anglers fishing the lake. In other words, it is assumed that an unbiased estimate of the true average results from the sample selected. Strict random sampling, however, will not necessarily give the most useful estimates in terms of low sampling error. Although the estimates so obtained may be unbiased they might be highly erratic, especially when the number of anglers contacted is low. Whenever deviation from random sampling is undertaken the method should be examined for sources of bias.

Three separate questions will be taken up in the following sections concerning estimates of average catch per hour per angler.

(1) What is the most efficient sampling method, or schedule for contacting anglers?

(2) How many angler contacts are necessary for a given precision of estimation?

(3) How should data be analysed to adjust for disproportionate sampling?

In addition to these questions the precision actually attained in the Intensive Creel Census on several lakes will be determined.

Comparison of Efficiency and Sample Size Required for Several Sampling Methods, Bear Lake, 1941

Required Number of Angler Contacts Assuming Random Sampling. The number of angler contacts necessary for estimates of the average catch per hour per angler depends on, among other things, the variability of the individual catch-per-hour values encountered. It has been noted already in this study that catch data are extremely variable and this variability may be a limiting factor in the application of sampling techniques to creel census studies.

It is possible to estimate the number of angler contacts that will be necessary for an actual census if some advance information is available about the variability of the catch data. From the relationship

$$t = \frac{\bar{x} - m}{\frac{S}{\sqrt{N}}} \qquad \text{(Snedecor, 1948: p. 457)}$$

the sample size required for a given precision and confidence level may be estimated. This relationship, rearranged and solved for N, is

$$N = \frac{t^2 S^2}{(\bar{x} - m)^2}$$

where $(\bar{x} - m)$ is the maximum difference between the sample mean and the corresponding population parameter m that will be tolerated in the study at hand, and S^2 is an estimate of the population variance (σ^2) from previous samples. For a given precision of estimating and at any significance level (e.g., t value equal to 1.96 at 5% level) the required N will increase or decrease with S^2 (Figure 5). In this procedure it is assumed that the distribution of means is approximately normal. The validity of this assumption will be discussed in a subsequent section.

Figure 5. Relationship between the Required Number of Angler Contacts and the Variance of Catch-per-Hour Data for Various Limits of Precision and at the 95 Percent Confidence Level

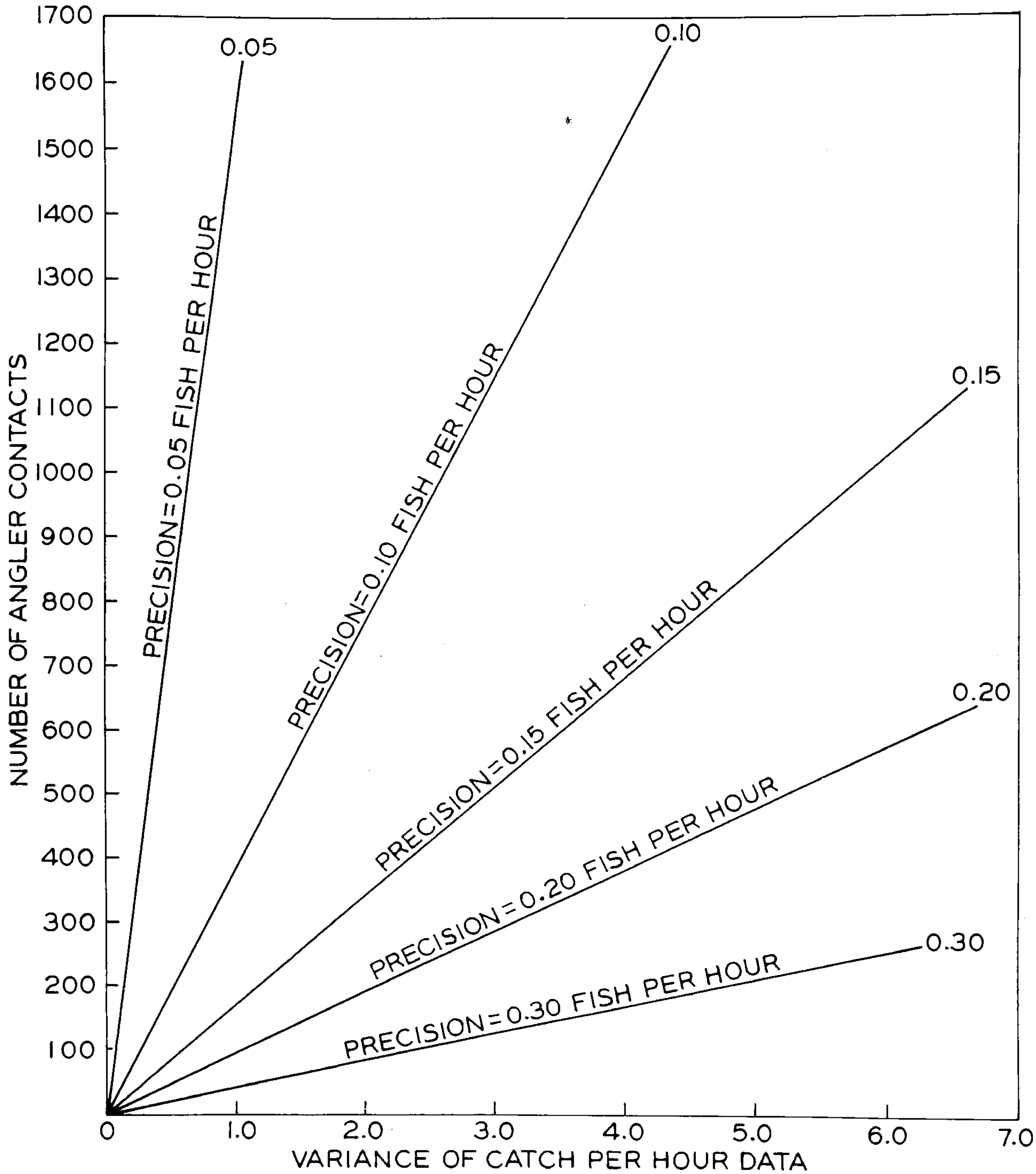

1700
1600
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
NUMBER OF ANGLER CONTACTS
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
VARIANCE OF CATCH PER HOUR DATA
0.05
PRECISION=0.05 FISH PER HOUR
0.10
PRECISION=0.10 FISH PER HOUR
0.15
PRECISION=0.15 FISH PER HOUR
0.20
PRECISION=0.20 FISH PER HOUR
0.30
PRECISION=0.30 FISH PER HOUR

Figure 5.

An advance estimate of the variance may be available from earlier census work or from preliminary records gathered at the study area.

An estimate for Bear Lake of the population variance (σ^2) is available from the variance (S^2) of the individual catch records. Appropriate values were inserted in the familiar formula for variance as follows:

$$S^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{N}}{N - 1}$$

$$S^2 = \frac{4808 - \frac{(1683)^2}{1814}}{1813} = 1.79$$

Therefore at the 99 percent confidence level and a precision of 0.1 fish per hour the required number of angler contacts is

$$N = \frac{(2.6)^2(1.79)}{(0.1)^2} = 1210$$

At the 95 percent confidence level this required sample size reduces according to $N = 688$, and if one were to be satisfied with a precision of ±0.2 fish per hour, at the 95 percent confidence level, the sample size is further reduced to $N = 172$.

It is apparent that a large proportion of the anglers must be contacted to attain a precision of 0.1 fish per hour if the sampling approach is strict random sampling. Two reasons prompted investigation of this variance and the rather large required sample size: (1) some increase in efficiency of sampling may be possible if the components of this total variation are considered; and (2) random selection of a sample from the total population of anglers is not a practical field technique.

Correction for Finite Population. Cochran (1953) suggested a modification of technique in estimating the required sample size where the sample is a large fraction of a finite population. Cochran's method was applied as follows to the Bear Lake 1941 data for a more precise estimate of the standard error of the mean catch per hour and required sample size. N is the required number of angler contacts after correction of an initial estimate N_o.

$$N = \frac{N_o}{1 + \frac{N_o}{N}} \qquad \text{(Cochran, 1953: p. 56)}$$

$$N = \frac{688}{1 + \frac{688}{1814}} = 499$$

This is a considerable reduction in the sample size required for the 95-percent-0.1-fish levels assuming random sampling.

The Required Number of Angler Contacts Assuming Stratified Random Sampling. A further increase in efficiency in sampling, i.e., more accurate estimates for a given amount of census effort and/or money, is possible if the population of anglers can be separated into categories or strata which are more homogeneous than the entire population. Anglers fishing on a lake are naturally stratified by days, and also may be separated into various periods within days. If certain days are more or less homogeneous with respect to catch per hour per angler than are other days of the week it might be advantageous to sample more or less often on such days. The days on which the fishing intensity is unusually high are suspected of having more variable mean catches than days of low intensity. Usually, more fishermen appear on lakes on week-end days and holidays than on weekdays. The complete census data for Bear Lake in 1941 were analysed for variation between days and within days (Table 2). The number of angler contacts required during the summer season assuming random sampling, and after correction for a finite population, is 499 (from Cochran's formula, see above).

There are 7 days (Monday through Sunday) with 11 or 12 of each occurring during the season. There are certain days on which heavy concentrations of anglers usually appear: opening day of the season; 4th of July; and Labor Day. These days are treated separately here to learn if they should be sampled more or less heavily than ordinary days.

Two methods of allocating the required 499 contacts among the 8 types of days will now be considered--equal allocation, and allocation in proportion to both size and variability of strata.

Equal Allocation of Total Sample. With equal allocation a total of approximately 63 contacts would be made on each type of day (499/8 = 63+). The following computations concern the standard error of the mean and confidence limits of the mean for the 1941 Bear Lake catch-per-hour-per-angler data after stratification by days. All catch-per-hour data were analyzed separately for each type of day to determine the mean and variance of the respective strata. The variance of the total mean and the confidence limits for mean catch per hour (Table 4) were calculated by the following methods:

Confidence limits: $\bar{y} \pm ts_{\bar{y}}$

$$\text{Variance: } \frac{1}{N^2} \sum_{h=1}^{L} N_h (N_h - n_h) \frac{S_h^2}{n_h} \quad \text{(Cochran, 1953: p. 69)}$$

where N = the total population of 1814 angler records; N_h = the total number of angler records in the day_h stratum; n_h = the size of the sample from the day_h stratum; and S_h^2 = the variance of the day_h stratum.

Table 2. Stratification of Bear Lake, 1941 Catch-per-Hour-per-Angler Records by Days

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Holidays
Total Number of Anglers N_h	419	133	174	159	217	295	283	134
Average Catch per Hour per Angler	0.88	1.07	1.02	0.94	1.07	0.91	0.74	1.00
Variance	2.90	2.02	1.51	1.88	1.57	1.32	1.13	1.09

The resulting confidence limits of ±0.12 for stratified random sampling represent something of a loss of efficiency. Strict random sampling produced a more precise estimate for the same total sample of 499 contacts than did stratified random sampling with equal allocation among days.

Optimum Allocation of Total Sample. The merits of allocating the total sample among strata in proportion to the respective size and variability of each stratum were considered next. Development of this method of optimum allocation is credited to J. Neyman and is described by various authors (see Snedecor, 1948: p. 460). A further discussion and application of Neyman sampling is included in this paper in the section concerned with the allocation of boat counts. Allocation of the 499 angler contacts by this method concentrated more sampling effort on Sundays and reduced the total number of contacts to be made on certain weekdays and holidays (Table 3). The variance of the mean catch was calculated, again by Cochran's formula, and confidence limits of ±0.97 should be ±0.097 were computed for the mean (Table 4). This represents a 3 percent improvement for stratified random sampling with optimum allocation over strict random sampling.

These comparisons show only a small advantage for stratified sampling of the small, relatively homogeneous population of anglers such as were encountered on Bear Lake in 1941.

The advantage of stratifying catch-per-hour data from a lake on which there is considerable fishing intensity was examined next.

Pontiac Lake in Oakland County is fairly large (585 acres) and is located near a population center and thus receives heavy fishing pressure.

Table 3. Optimum Allocation of Total Number of Angler Contacts Among Days Bear Lake, 1941

Day		Total Number of Anglers N_h	Standard Deviation σ	$N_h \sigma$	Number of Contacts $n_h = N_h \sigma (n/S)$
Sunday	(1)	419	1.703	713.56	148
Monday	(2)	133	1.421	188.99	40
Tuesday	(3)	174	1.229	213.85	45
Wednesday	(4)	159	1.371	217.99	46
Thursday	(5)	217	1.253	271.90	57
Friday	(6)	295	1.149	338.96	71
Saturday	(7)	283	1.063	300.83	63
Holidays		134	1.044	139.90	29
Total for Season		1814		S = 2386	N = 499

Table 4. Comparison of Efficiency of Several Methods of Sampling to Estimate Average Catch per Hour per Angler. Bear Lake, 1941, Summer Season

	Strict Random	Stratified Random with Equal Allocation Among Days	Stratified Random with Optimum Allocation Among Days
Total Number of Contacts in Sample	499	499	499
$s_{\bar{x}}^2$	0.00260	0.00353	0.00248
$s_{\bar{x}}$	0.0510	0.0594	0.0497
Confidence Limits	±0.10	±0.12	±0.097
Mean Catch	0.93	0.93	0.93

Catch per hour records are available from this lake for several years, but the creel census has been incomplete and so only samples are available for analysis. A treatment parallel to the above Bear Lake study was accorded the data for the 1948 summer season with some changes. The modifications were that the strata variances (S_h^2) were estimates calculated from samples and that the total size of each stratum (N_h) was estimated from boat counts. These estimates may be biased or inaccurate to some extent, but the same values were used for all three sampling methods and are assumed to be comparable.

Size of Sample Required to Estimate Average Catch per Hour per Angler, Pontiac Lake, 1948, Assuming Random Sampling

An estimate of the required number of angler contacts is again available from the relationship of N and S^2.

$$N = \frac{t^2 S^2}{(\bar{x} - m)^2} = \frac{(3.84)(1.66)}{0.01} = 637.44$$

That is, if the variance ($S^2 = 1.66$) of catch-per-hour-per-angler records that were collected is taken as an estimate of population variance, and if it is assumed that all anglers have the same chance of appearing in the sample, then the required sample size is 637, to obtain a precision of ±0.1 fish per hour at the 95 percent confidence level. The mean catch per hour per angler in this example is 1.01 fish. The total number of anglers is estimated to be 41,200 (Table 5). With this large population the sampling ratio is negligible and the correction for finite population is not applicable.

Table 5. Computation of Estimates of Total Fishing Intensity by Days, Pontiac Lake, 1948

	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Total
(1) Average Number of Boats per Hour	120	35	25	33	17	49	80	
(2) Average Number of Anglers per Boat	2.37	1.92	2.36	2.19	1.83	2.51	2.47	
(3) Average Number of Anglers per Hour (1) X (2)	284.4	67.2	59.0	72.3	31.1	123.0	197.6	
(4) Angler Hours per Day (3) X 15.5	4,408	1,042	915	1,121	482	1,907	3,063	
(5) Hours per Angler per Day	4.1	4.4	1.5	4.1	3.0	4.8	4.5	
(6) Estimate of Number of Anglers for Season (4)/(5) X 12	12,900	2,842	7,320	3,281	1,927	4,767	8,168	41,205

Efficiency of Stratified Sampling by Days, Pontiac Lake, 1948

Estimates of the total number of anglers fishing each type of day were derived from Intensive Creel Census records (Tables 5 and 6). These values for N_h were substituted in the formula for variance of the mean for stratified random sampling.

$$\text{Var. } \bar{y} = \frac{1}{N^2} \sum_{h=1}^{L} N_h (N_h - n_h) \frac{s_h^2}{n_h} \qquad \text{(Cochran, 1953: p. 72)}$$

The 637 contacts, which is the number necessary for random sampling, were first allocated equally among the various days of the week. For example, a total of 91 contacts would be made on the Sundays during the season. Thus, the n_h values are equal in this application of the formula. The value obtained for standard error of the mean catch by this allocation is 0.0602, the confidence limits are ±0.12. This is poorer precision than the ±0.10 specified for strict random sampling. However, the apparent loss of efficiency for stratified sampling with equal allocation may not be real because the original sample was secured more by stratified than by strict random sampling.

Optimum Allocation of Contacts by Days, Pontiac Lake, 1948

The total sample of 637 contacts was allocated among the days in proportion to the total number of anglers and the standard deviation of the catch-per-hour-per-angler records (Table 7). These new values for n_h were substituted in the formula for variance of the mean of stratified random sampling with the following result.

$$\text{Var. } \bar{y}_{st} = 0.00229$$

$$\text{Standard error } \bar{y}_{st} = 0.04785$$

$$\text{Confidence limits } = \pm 0.095$$

Table 6. Stratification of Catch-per-Hour-per-Angler Records by Days. Pontiac Lake, 1948

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Total
Estimated Total Number of Anglers	12,900	2,800	7,300	3,300	1,900	4,800	8,200	41,200
Number of Anglers Contacted	208	66	117	118	28	200	232	969
Optimum Allocation	305	81	100	75	23	126	214	969
Average Catch per Hour per Angler	1.21	1.02	0.60	1.03	0.43	0.98	0.92	0.96
Variance	1.97	2.23	0.51	1.40	0.37	1.85	1.82	1.66

Table 7. Optimum Allocation of Angler Contacts by Days, Pontiac Lake 1948

Day	N_h	σ	$N_h\sigma$	* $n_h = N_h\sigma(n/S)$				
Sunday	12,900	1.40	18,060	203	216	230	308	350
Monday	2,800	1.49	4,172	47	50	53	71	81
Tuesday	7,300	0.71	5,183	58	62	66	88	100
Wednesday	3,300	1.18	3,894	44	47	50	66	75
Thursday	1,900	0.60	1,140	13	14	14	20	23
Friday	4,800	1.36	6,528	73	78	83	111	126
Saturday	8,200	1.35	11,070	124	133	141	189	214
	N = 41,200		S=50,047	n=562	600	637	853	969

Precision Actually Attained in Estimating the Average Catch per Hour per Angler in the Intensive Creel Census of Pontiac Lake during the Summer Season of 1948. A total of 969 anglers were interviewed for catch information on Pontiac Lake during the summer season. The schedule for the census allowed about equal sampling on the different types of days, but there was a tendency for the technicians to make more contacts on days when many anglers were present. The distribution of those contacts was thus somewhat in proportion to the fishing intensity, and the over-all result of the schedule used was a nearly optimum allocation of the 969 contacts among days.

An estimate of the precision actually attained was calculated from Cochran's formula for variance of the mean of stratified random sampling. For comparison the total sample of 969 contacts was stratified among days with optimum allocation (Table 7), and the variance of the mean and confidence limits were recalculated (Table 8).

Table 8. Comparison of Precision Actually Attained in 1948 in Estimating Average Catch per Hour per Angler and the Precision Possible by Optimum Allocation

	$s_{\bar{y}}^2$	$s_{\bar{y}}$	Confidence limits
Actual schedule of contacts	0.00172	0.04147	±0.081
Optimum allocation of contacts	0.00157	0.03962	±0.078

The significance of these figures is, that although the sampling schedule in 1948 tended toward optimum allocation and was more than adequate to attain a precision of ±0.10 fish per hour (it actually was ±0.081), still greater precision would have been possible if exactly optimum allocation of the 969 contacts had occurred.

Amount of Reduction of Sample Size Equivalent to the Reduction in Confidence Limits Attained by Stratified Sampling with Optimum Allocation

The difference of 6 percent in the confidence limits of the mean (0.100-0.094), as determined for stratified sampling with allocation and for random sampling, represents a gain in efficiency. This reduction in confidence limits should be translated into an equivalent reduction in sample size when the precision is fixed at ±0.10. The equation for sample size is

$$N = \frac{t^2 S^2}{(\bar{x} - m)^2}$$

where t and $(\bar{x} - m)^2$ are fixed values, and S^2 is computed from the sample. The S^2 value 1.66 for Pontiac Lake, 1948, was determined from records collected more or less at random, and the required N calculated as 637. However, a new estimate of the variance is available after stratification. The variance of the mean for stratified sampling is 0.00229. From the relationship of the variance of the mean and the variance of the population $\left(S_{\bar{y}}^2 = \frac{S^2}{N}\right)$, S^2 becomes

$$S^2 = NS_{\bar{y}}^2 = 637 \times 0.00229 = 1.46$$

This new estimate of S^2 is substituted in the formula for sample size.

$$N = \frac{t^2 s^2}{(\bar{x} - m)^2} = \frac{3.84 \times 1.46}{(0.1)^2} = 561 \text{ (approx.)}$$

As a check on the adequacy of 561 contacts the sample was reallocated among the various strata as before, (Table 7) and the variance of the mean recalculated by Cochran's formula for stratified sampling with the following results.

$$s_{\bar{y}}^2 = 0.00260$$

$$s_{\bar{y}} = 0.051$$

$$\text{Confidence limits} = \pm 0.10$$

Thus, for a fixed precision of ±0.10, the number of angler contacts required to determine the mean catch per hour is reduced from 637 to 561 when stratified random sampling with optimum allocation is assumed rather than strict random sampling.

<u>Effects of Non-Normality of Catch-per-Hour Data on Confidence Limits of Mean and Estimates of the Variance</u>

In census work it is usually assumed that "normal statistics" are applicable to the statistical treatment of mean values, such as determining the standard error and confidence limits of the mean. A basic concept of statistics is that if a population is normally distributed with variance σ^2, the distribution of means of random samples of size n from that population will be normally distributed with a variance of $\frac{\sigma^2}{n}$. The distribution of catch-per-hour data is not normal, however. It exhibits a marked positive skewness (Figure 6). The assumption of normality is used primarily to calculate confidence limits. A statement that the confidence limits are $\bar{y} \pm 1.96\, s_{\bar{y}}$ considers the deviation of

Figure 6. Frequency Distribution of Catch-per-Hour-per-Angler

Data from Bear Lake, 1941

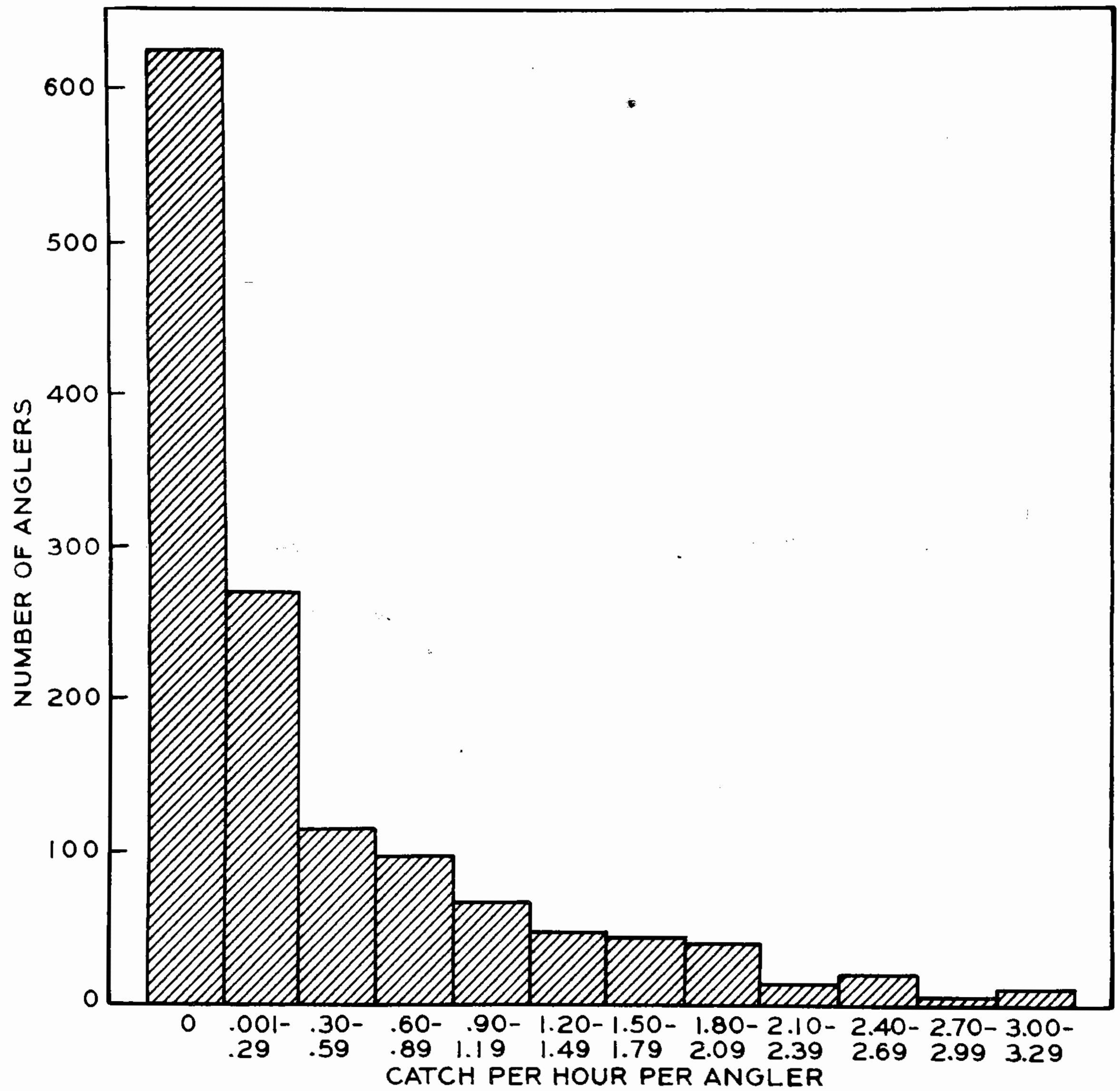

600
500
400
300
200
100
0
NUMBER OF ANGLERS
0
.001- .29
.30- .59
.60- .89
.90- 1.19
1.20- 1.49
1.50- 1.79
1.80- 2.09
2.10- 2.39
2.40- 2.69
2.70- 2.99
3.00- 3.29
CATCH PER HOUR PER ANGLER

Figure 6.

sample mean values about the true population mean. If the mean values are not normally distributed the variability of the mean will not be accurately described by its standard error.

Fortunately, it has been well established that the distribution of sample means tends to normality as the size of the sample increases. (see, for example, Kenney and Keeping, 1951: p. 144). It is important, however, that the size of the sample of catch records required for use of the normal law theory be established. Cochran concludes that for populations with positive skewness the sample should be at least $n > 25G_1^2$, where G_1 is a measure of skewness.

$$G_1 = \frac{1}{N\sigma^3} \sum_{i=1}^{N} (y_i - \bar{Y})^3 \quad \text{(Cochran, 1953: p. 25)}$$

For the Bear Lake (1941) data $G_1 = 1.6$, and N was computed as 64. This means that at least 64 catch-per-hour records must be included in the sample for assurance that the normal law theory is applicable. The method used here for computing the skewness factor was suggested to me by H. Carver, and is included (Appendix B). Further discussion of this question is presented in a subsequent section on boat count procedures.

However, in statistical analyses involving probability statements about the parent distribution the degree of skewness must be considered. If, for example, an estimate is desired of the number of anglers falling within a specified range on either side of the mean catch per hour special methods must be used. In a normally distributed sample the data is symmetrically grouped about the mean, and the interval $\bar{X} \pm 2s$ includes 95 percent of the data ($\bar{X}$ = mean and s = standard deviation).

To estimate the number of variates within a specified interval of the mean one need only reduce the variates to standard units ($t = \frac{\bar{X} - m}{\sigma}$) and refer to a table of the normal distribution in standard units (t - Table of normal areas and ordinates).

A different treatment is necessary for skew distributions such as the distribution of the frequencies of different catch-per-hour values. This distribution is not symmetrical and the number of anglers who have less than the mean catch rate is not equal to the number of anglers who have more than the mean rate. In this case considerable error would result if tables of normal areas were used. Instead, tables of areas of Pearson's Type III Frequency Function should be referred to at the appropriate level of skewness and value for t. (see Statistical Tables by Harry C. Carver, 1940).

The effect of non-normality of the parent distribution of catch records on estimates of variance is not as well known. Estimates of the variance are used in two ways in this study: (1) to test whether catch values and boat counts on certain days or hours are more variable than on others; (2) to compare precision of estimating obtained by two methods--strict random sampling and stratified sampling. The variances of the catch records and counts of different days or hours will be the basis for allocating the sampling effort among days or hours. In these studies little difficulty is anticipated because of errors in estimating the variance. This is true because the same estimates of variance are used in a comparative manner. However, in other applications some idea of the precision of estimates of the variances may be needed. Cochran (1953: p. 27) stated, "One effect of non-normality is that the estimated variance s^2 may be more highly variable from sample to sample than we expect if we assume that we are sampling from a normal distribution." Further discussion is beyond the scope of this paper. The reader is referred to Cochran (1953: p. 28) for a more detailed explanation. The conclusion of that author is that the precision of estimates of variance should be considered if that variance is based on rather scanty data. In creel studies the number of records is usually large and probably no important error is introduced in estimates of variance by sampling from a nod-normal parent population.

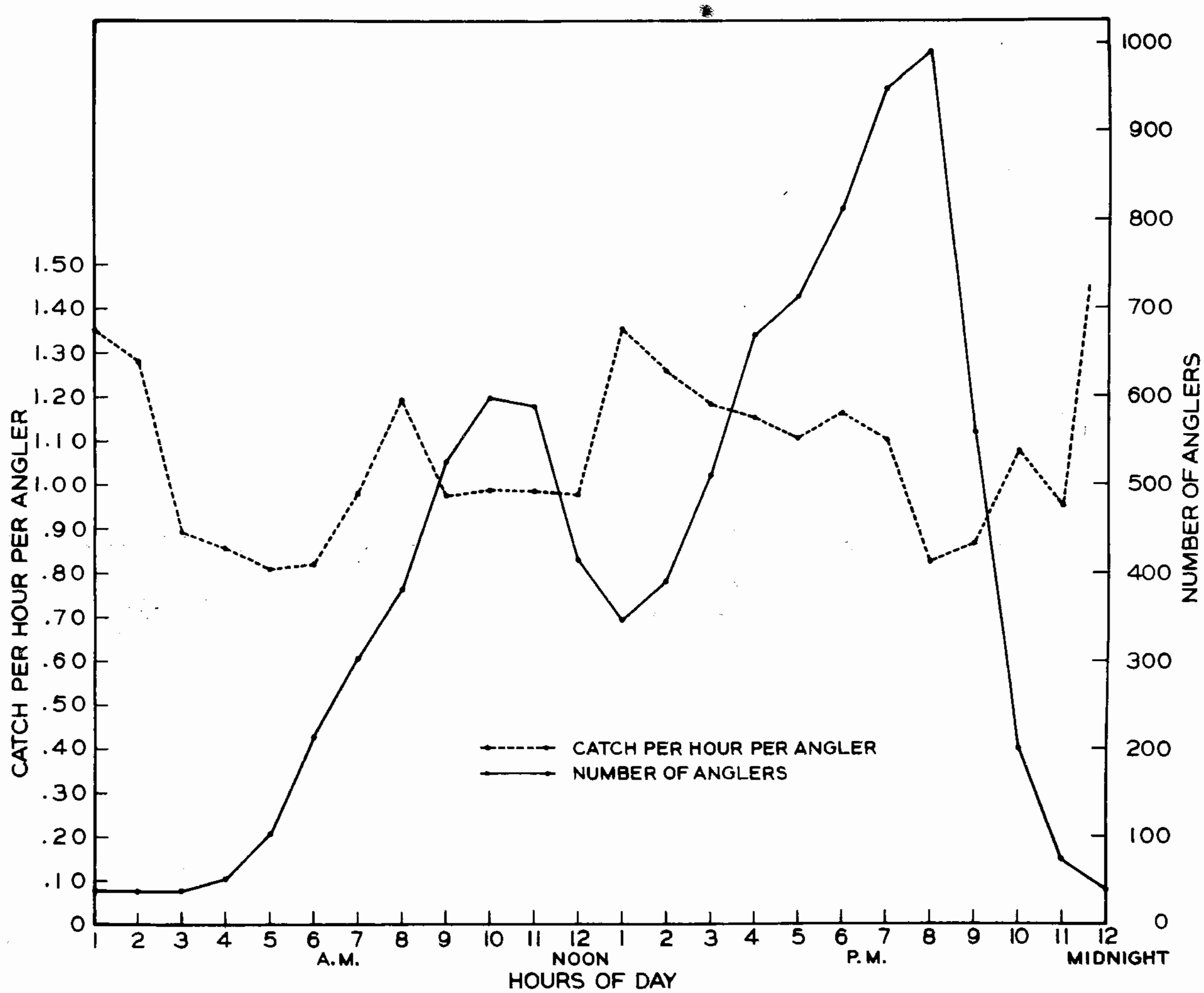
The Effect of Disproportionate Sampling Ratios on Methods of Analyzing Data

An average catch-per-hour value, such as a seasonal average, contains records from periods which are of varying fishing quality and fishing intensity. Some thought must be given to the size of samples obtained during each period in relation to the number of anglers, and to the method of combining records if arithmetic mistakes in calculating averages are to be avoided.

Fishing quality varies during the day, and the intensity of anglers on the lake changes considerably (Figure 7). Catch-per-hour-per-angler values may vary in a consistent manner, with perhaps a high value at a certain time, or may simply be erratic. Typically, the curve of fishing intensity rises in the morning decreases about midday, and reaches a maximum in the evening hours. Thus, each hour during the day is not of equal importance in calculating the average catch per hour. A simple example with hypothetical data may serve to illustrate this.

Suppose that the average catch per hour per angler is desired for one day, and further, suppose that the catch-per-hour values of the few anglers on the lake in the morning are significantly different from values in the evening when many more anglers are out (Table 9).

Figure 7. Total Number of Anglers Fishing at Various Hours and the Average Catch Rate during the Summer Season, Bear Lake, 1941



CATCH PER HOUR PER ANGLER
NUMBER OF ANGLERS
HOURS OF DAY
A.M.
NOON
P.M.
MIDNIGHT
0
.10
.20
.30
.40
.50
.60
.70
.80
.90
1.00
1.10
1.20
1.30
1.40
1.50
100
200
300
400
500
600
700
800
900
1000
1 2 3 4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12

Figure 7.

Table 9. Hypothetical Catch-per-Hour Values of All Anglers for One Day

	Morning Period	Evening Period
	1.0	2.0
	1.0	2.0
	1.0	2.0
	1.0	2.0
	1.0	2.0
	1.0	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
	...	2.0
$\bar{X}$ =	1.0	2.0
N =	6	18

The true average of all 24 anglers in Table 9 is $\frac{(18 \times 2) + (6 \times 1)}{24} = 1.75$; it is not the average of two averages, i.e., $\frac{1.0 + 2.0}{2} = 1.5$. This seems too obvious to mention and yet it is easy to overlook this simple concept in compiling census data. Suppose that instead of recording all anglers that fished on the lake during the day only a sample was taken in the morning and another in the evening. Samples taken from these homogeneous populations will yield adequate and unbiased estimates of the true mean regardless of size. These values might appear in the samples selected:

									$\bar{X}$	N
Morning period	1.0	1.0	1.0	1.0					1.0	4
Evening period	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	8

The true average for the day as noted above is 1.75 and not

$$\frac{(4 \times 1.0) + (8 \times 2.0)}{12} = \frac{20}{12} = 1.66$$

The sample averages, weighted by the respective number of anglers each period yields the true average, however, which is

$$\frac{\frac{4 \times 1.0}{4}(6) + \frac{8 \times 2.0}{8}(18)}{24} = 1.75.$$

The same result, of course, ensues if the size of each sample is proportionate to the total number of anglers each period, and the simple average of all records is calculated. For example, if a morning sample of $N_1 = 3$ or 50 percent of the total of 6 anglers, and an evening sample of $N_2 = 9$, which is 50 percent of 18 anglers, is selected, the true average is attained.

$$\frac{(3 \times 1.0) + (9 \times 2.0)}{12} = 1.75.$$

This example has been oversimplified in that the question of adequate sample size is not considered. Actual field data are much more variable.

In practice, the effects of sampling and the need for weighting averages are not so obvious as in the above example. Several factors must be operating concurrently for appreciable bias to be introduced into computations of average catch per hour per angler. These factors and the conditions under which they will produce bias in estimates are:

(1) VARYING FISHING INTENSITY. The number of anglers must change markedly during periods of the day.

(2) CHANGING CATCH PER HOUR PER ANGLER. There must be a significant difference in the recorded fishing quality (catch per hour per angler) between periods of differing fishing intensity.

(3) DISPROPORTIONATE SAMPLE SIZE. The size of samples drawn from the periods of differing fishing intensity must fail to be in proportion to the total fishing intensity each period.

Clearly, fishing intensity varies during the day. The distribution of anglers by hours as obtained for Bear Lake (Figure 7) is found to represent conditions on many lakes in Michigan. Eschmeyer (1936) found essentially the same pattern of fishing on Fife Lake.

The fluctuations in fishing quality (catch per hour per angler), however, are not so distinct, or as predictable. The Bear Lake catch-per-hour data for 1941 show an erratic behavior between hours, but with a distinct difference between the average for the morning period and the average for the evening period.

The number of anglers contacted may or may not be in proportion to the total number of anglers present. On lakes which are lightly fished, such as Bear Lake, the sampling ratio obtained seems to be nearly constant. On large, heavily fished lakes sampling is conducted from a wide range of fishing intensities and the ratios obtained are not constant, but decrease when large numbers of anglers are present (as in Figure 11). This is true because census clerks are unable to increase the number of contacts beyond a certain level. The sampling ratios actually obtained seem to vary with the lake in question. The physical character of the body of water, the development of resorts, cottages and liveries, and the attitude and behavior of anglers toward the census operation, all contribute to the ease or difficulty of obtaining census records. Sampling ratios are also affected by variation in the efficiency of clerks. Some obtain more records than others under conditions which are apparently identical (Figure 9).

The Intensive Creel Census has been conducted on twelve lakes in Michigan over a period of eight years. Considerable data are thus available which indicate the number of angler interviews and the sampling ratios that census clerks have obtained under different conditions. This information provides a practical basis for setting up schedules, and secondarily it is a means of evaluating the work performance of the clerks in the field.

Boat count records and the slips on which angler contacts are recorded were analyzed for Bear and Pontiac lakes for the years 1947 to 1952. The number of boats that were included in the angler contact slips for each day was tallied by hours of the day. Thus, on the hours on which counts were made of boats present on the lake, corresponding information was available about the number of those boats that appeared in the angler contacts. This analysis was facilitated by the method of recording data. All angler contact information for one boat was combined on one record slip. This ratio of the number of boats contacted to the number of boats counted is defined as the sampling ratio in this study.

Example of the Sampling Ratios Obtained on a Lake with Low Fishing Intensity and the Difference between Weighted and Unweighted Calculations of Catch per Hour per Angler. The present method of conducting the Intensive Creel Census at Bear Lake has resulted in a fairly constant sampling ratio. The result is that little error is introduced in the computation of average catch per hour per angler by a simple arithmetic average of all records collected. A comparison of complete census data for Bear Lake in 1941 and subsamples drawn therefrom by the method of the Intensive Census provides a measure of the adequacy of the method. Before such a comparison can be made, however, it is necessary to establish the size of samples that could be expected to be drawn from such a population of records. No sampling program was in operation at the time the complete census was conducted. As an alternative, the sampling ratios actually obtained on this lake in six later years when a sampling program was operative, provide a basis for subsampling the earlier complete data (Figure 8). It is assumed in this analysis that the sampling ratios established for boats holds for anglers also. The average subsample size expected for a given intensity of anglers was interpolated for the 1941 data from this curve (Columns 5 and 6, Table 10). The additional data of Table 10 are a breakdown of numbers of fishermen and their recorded catch per hour by hours of the day. For example, if an angler fished from 8:00 a.m. to 10:00 a.m. he

Figure 8. Six-Year Average Ratios of Boats Contacted to Boats Counted, Bear Lake

Figure 9. Comparison of Average Ratios of Boats Contacted to Boats Counted Obtained by Two Census Clerks on Bear Lake

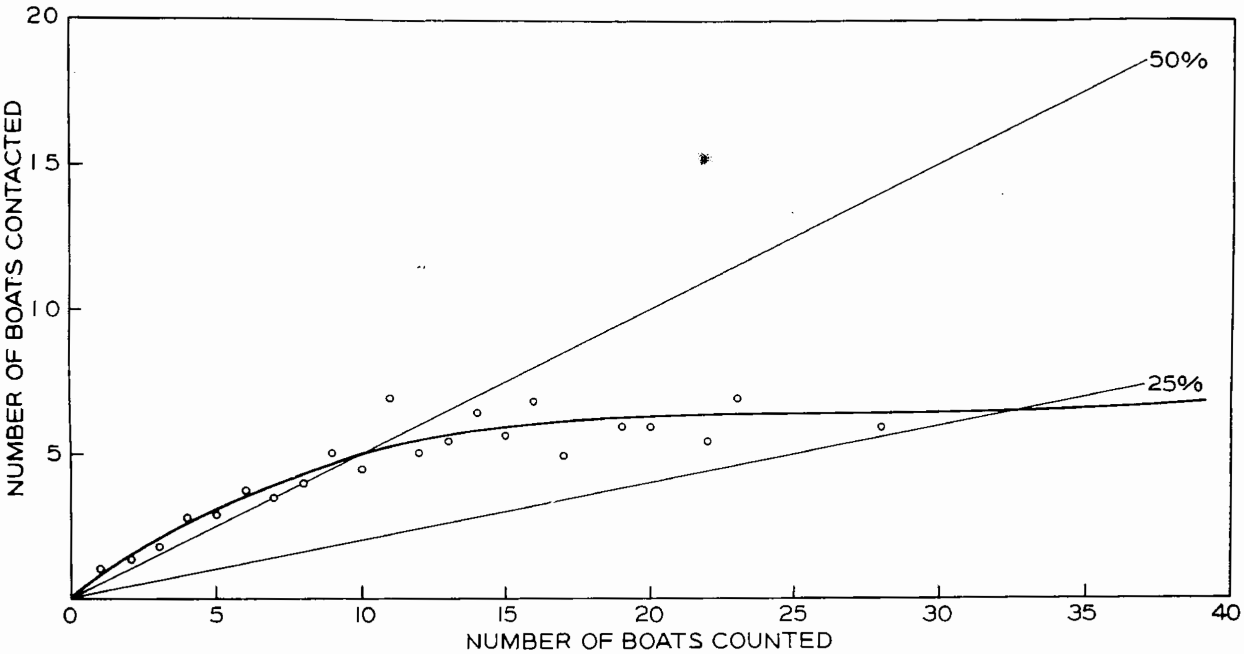


Figure 8.

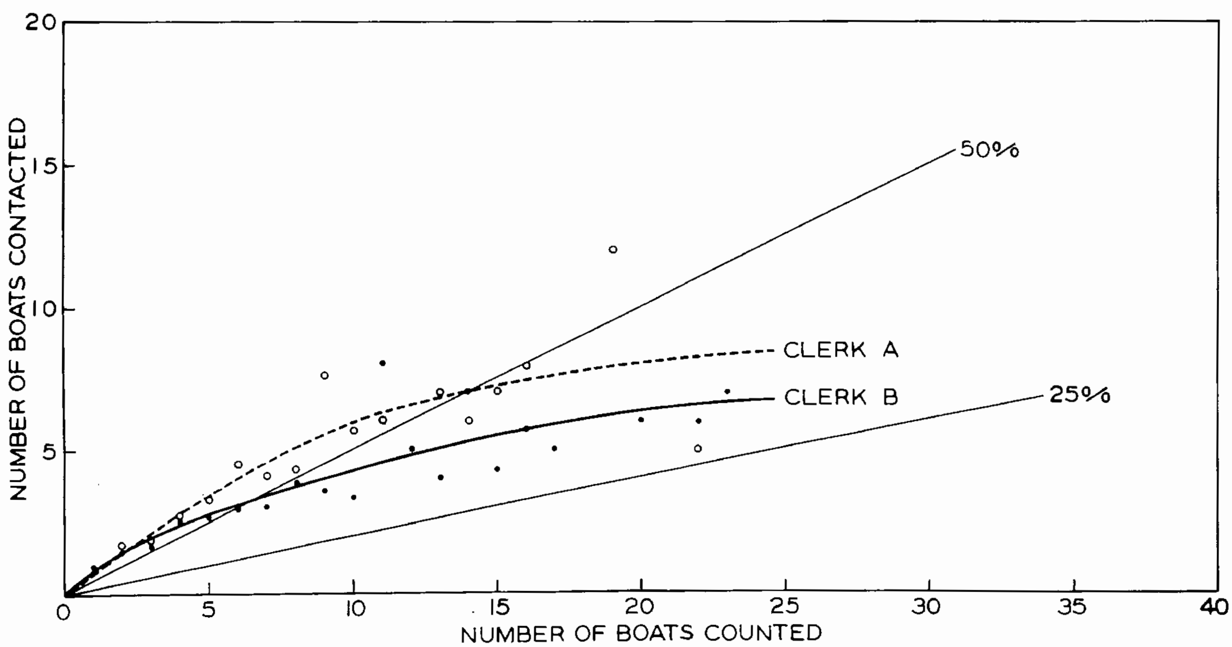


Figure 9.

Table 10. Sample Size Selected for Bear Lake, 1941. Data Based on Sampling Ratios Actually Attained In Six Later Years

	Complete Census Data			Sample Data	
Time Recorded	Total Number Angler Hours Recorded for Season	Average Number Angler Hours per Day	Average Catch per Hour per Angler	Total Number Angler Hours for Season Expected in Sample	Average Number Angler Hours per Day Expected in Sample
Col. 1	Col. 2	Col. 3	Col. 4	Col. 5	Col. 6
8 a.m.	381	3.12	1.20	268	2.2
9 a.m.	525	4.3	0.97	342	2.8
10 a.m.	597	4.89	0.99	366	3.0
11 a.m.	589	4.83	0.98	366	3.0
12 Noon	413	3.39	0.97	280	2.3
1 p.m.	345	2.83	1.35	256	2.1
2 p.m.	388	3.18	1.25	268	2.2
3 p.m.	510	4.18	1.19	329	2.7
4 p.m.	667	5.47	1.15	403	3.3
5 p.m.	713	5.84	1.10	427	3.5
6 p.m.	813	6.66	1.16	464	3.8
7 p.m.	945	7.75	1.10	525	4.3
8 p.m.	989	8.11	0.83	537	4.4
9 p.m.	559	4.58	0.86	354	2.9

appeared in the census at 8, 9, and 10 o'clock. The true catch per hour of the complete data is computed in the following manner (from columns 1, 2, and 4, Table 10):

$$\frac{(381)(1.20) + (525)(0.97) + \ldots. + (559)(0.86)}{381 + 525 + \ldots. + 559} = \frac{8928.5}{8434} = 1.06$$

From columns 1, 5, and 4 of Table 10 the average catch per hour computed from samples drawn each hour is

$$\frac{(268)(1.20) + (342)(0.97) + \ldots. + (354)(0.86)}{268 + 342 + \ldots. + 354} = \frac{5517.1}{5185} = 1.06$$

This close agreement between the true average and the subsample average is almost too good, but it illustrates the effect of sampling in proportion to the number of anglers present (Figure 10). This result was not expected from a preliminary discussion of the problem when it was thought that census clerks would not be able to contact a fixed proportion of the anglers as more and more and more appeared on the lake. Actually, the clerks are not able to maintain a constant sampling ratio on more heavily fished lakes.

Example of the Sampling Ratios Obtained on a Lake with High Fishing Intensity and the Difference between Weighted and Unweighted Calculations of Average Catch per Hour per Angler. Pontiac Lake was heavily fished and more catch records were collected at this lake than at Bear Lake. The sampling ratio curve, however, is curvilinear over part of the wide range of intensities encountered (Figure 11). Clearly, the census clerks were not able to sample in proportion to the number of anglers

Figure 10. Average Number of Boats Contacted and Boats Counted by Hours, Bear Lake, 1947 to 1952

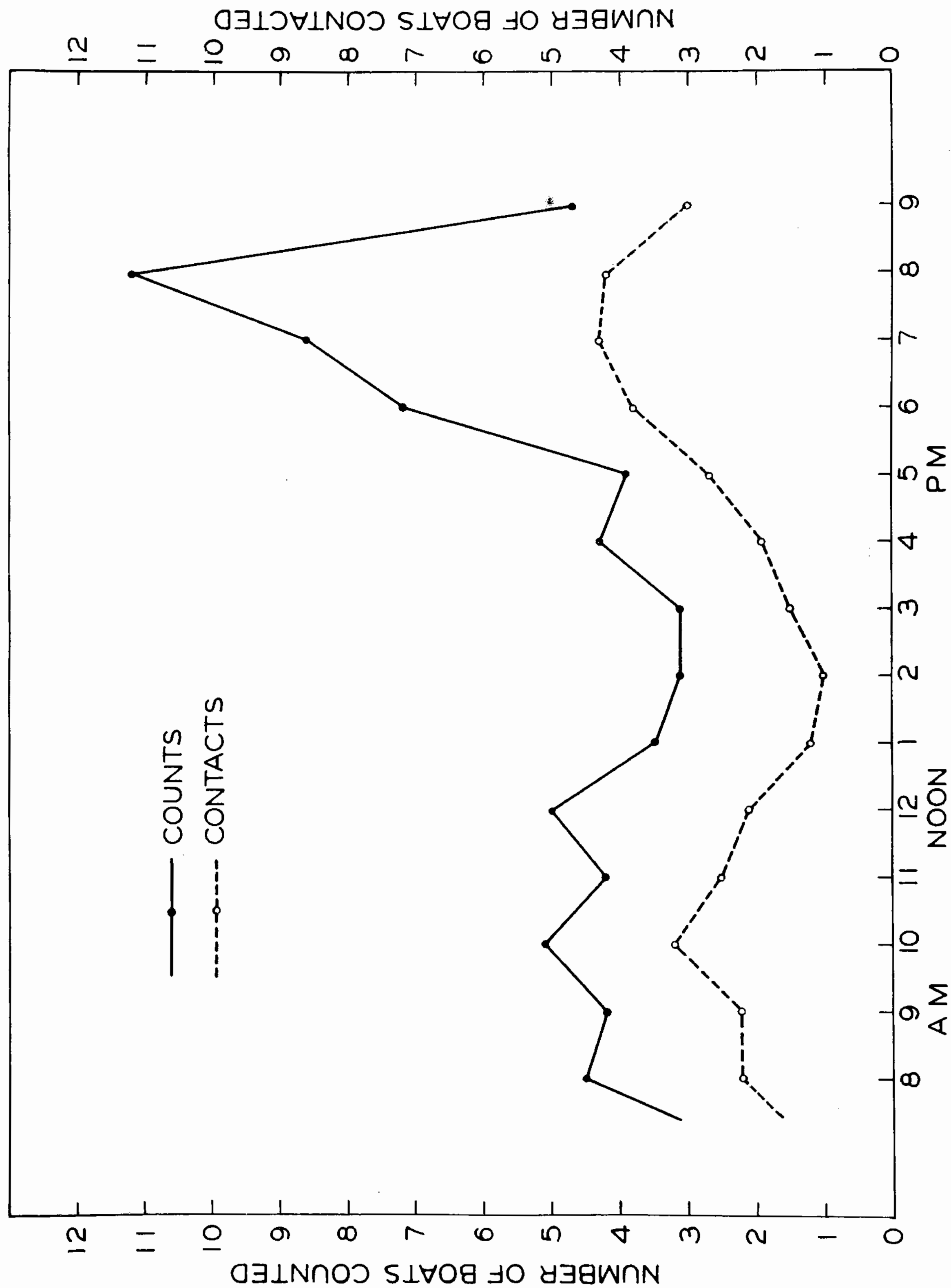

NUMBER OF BOATS CONTACTED
NUMBER OF BOATS COUNTED
COUNTS
CONTACTS
AM
NOON
PM

Figure 10.

Figure 11. Average Sampling Ratios of Boats Contacted to Boats Counted, Pontiac Lake, 1947 to 1952

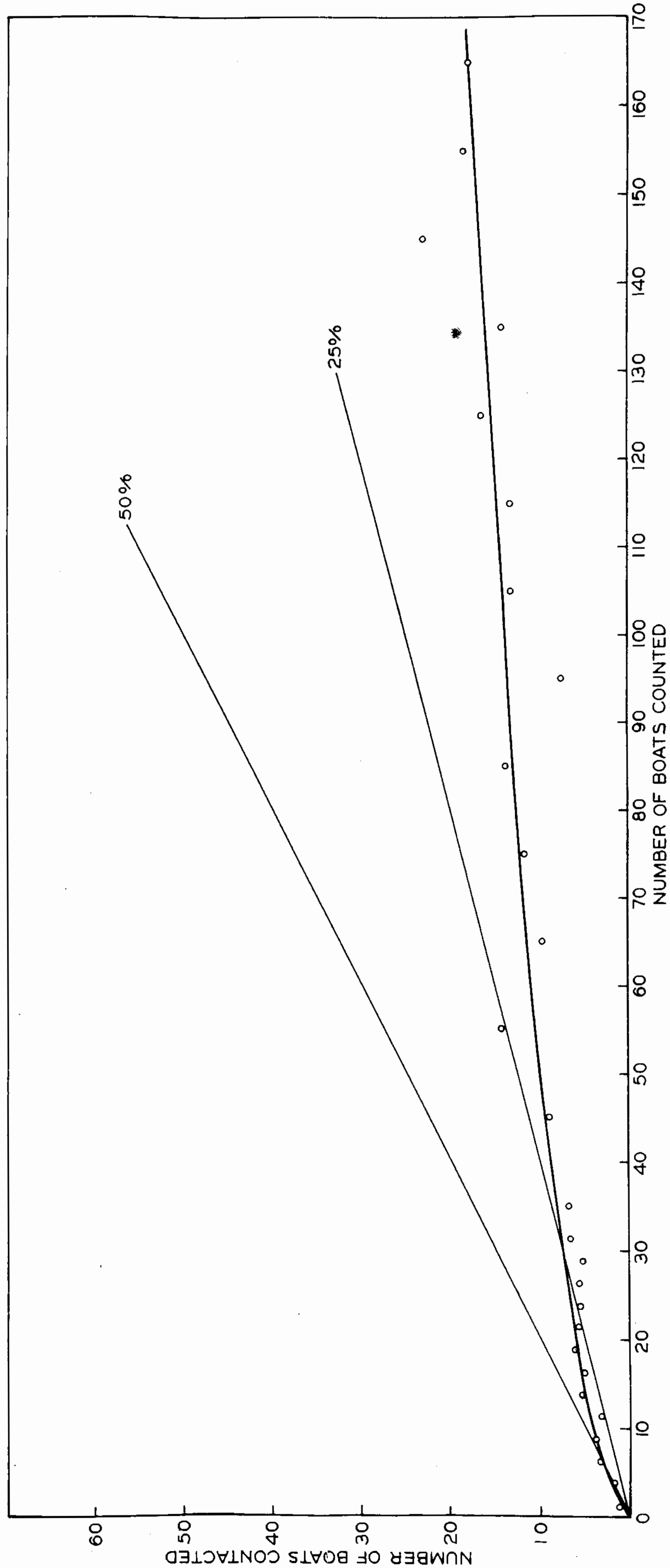

NUMBER OF BOATS COUNTED
NUMBER OF BOATS CONTACTED
50%
25%
0 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 170
10 20 30 40 50 60

Figure 11.

present when the greater concentrations occur. Under these conditions considerable error can be introduced in computations of the average catch per hour per angler if the records from various periods are not weighted by the respective intensities of anglers.

It is assumed in this analysis that the samples collected for Pontiac Lake were adequate and unbiased, but the validity of this assumption does not affect the significance of the following results. This is a comparison of two methods of combining the same data. Once estimates of the average catch per hour, total intensity, and the sampling ratios have been made from the facts available, they must be used in the same manner as complete data.

Catch per hour data were assembled by hours for Pontiac Lake and then the average catch per hour per angler for the season was computed by two methods. In one of these trials, an unweighted average was obtained by the simple arithmetic averaging of all records collected; in the other, the weighted average was compiled by multiplying the average catch per hour of each hour by the number of boats counted during those hours. In this comparison differences between the weighted and unweighted averages ranging from 3 to 20 percent were observed (Table 11). It is clear that two separate values for the average of catch per hour are possible, and that some thought must be given to the method of analysing catch records if arithmetic mistakes are to be avoided. The need for weighting catch records has been demonstrated here through an analysis of the intensity of anglers and their catch by hours. Other occurrences of this error are possible. Records are collected from different days and combined for a daily average. Similarly, seasonal estimates are combined to form an annual estimate. The need for weighting samples should be considered whenever catch records are being combined.

Table 11. Comparison of Weighted and Unweighted Computations of Seasonal Average Catch per Hour per Angler for Pontiac Lakes. Weighted Values Were Obtained by Multiplying the Mean Catch per Hour per Angler of Each Hourly Period as Obtained from Contacts by the Number of Boats Counted on the Hour. Unweighted Values Were Obtained by Averaging the Catch Values Obtained Each Hour from Contacts

Year	Average Catch per Hour per Angler Unweighted for Intensity $\bar{X}_{uw}$	Average Catch per Hour per Angler Weighted for Intensity $\bar{X}_w$	Amount of Error $\bar{X}_w - \bar{X}_{uw}$	Percentage Error (Amount $\bar{X}_{uw}$ differs from $\bar{X}_w$)
1948	1.024	1.058	- 0.034	3.0
1949	0.814	0.765	+ 0.049	6.4
1950	0.787	0.994	- 0.207	20.8
1951	1.138	1.008	+ 0.130	12.8

Problems of Estimating Total Fishing Intensity

Method of Deriving Estimate of Total Angler Hours from Boat Counts

Fishing intensity, or number of hours spent fishing, reflects the popularity of a lake among anglers and is related, among other things, to the fishing quality. Furthermore, it is a useful factor in the computation of estimates of the total yield of fish. The procedure now used in the Intensive Census for estimating total angler hours combines information from contacts with anglers and data from boat counts. The numbers of boats on the lake are converted to total angler hours for the season in the following way.

$$(1)\quad \frac{\text{Number anglers in contacts}}{\text{Number boats in contacts}} \times \text{Average number boats per hour from counts} \times \text{Length of fishing day in hours} \times \text{Number of days in season} = \text{Total Angler Hours}$$

For any given hour of the day the product $\frac{\text{Number of anglers}}{\text{Number of boats}} \times$ Number of boats per hour is equivalent to the total number of angler hours. This expression obtains from the fact that an angler on the lake at one counting hour represents one hour of fishing. But, the same angler may appear in succeeding counts so the result of the foregoing equation (1) is number of angler hours and not number of anglers.

Assumptions in Method

The procedure for estimating total angler hours is the most complicated phase of the Intensive Creel Census in that many variables are involved. Each of the factors in equation (1) is an estimate derived from samples, except the number of days in the season, and each is therefore subject to sampling error.

The average number of anglers per boat is determined from information gained by the angler contacts and is the simple average number encountered in the boats during these angler contacts. There is, however, some possibility for error in the result caused by bias in the schedule for angler contacts.

The average number of boats counted at intervals is presumably equal to the average number of boat hours during the count interval, but some discrepancy may occur if the opportunity for a boat to be counted does not coincide with its elapsed time on the lake. This will depend, of course, on the counting schedule and on the techniques used.

Shore fishermen and anglers abroad during darkness are not counted. Although the amount of fishing in these hours is believed to be a relatively very minor part of the whole, it's quantity is not ascertained in the Michigan Intensive method.

These assumptions and questions will be considered in the following sections along with the important question of the number of boat counts required.

<u>The Length of Fishing Day</u>. The number of hours in the fishing day is an important factor in the calculation of total angler hours. In computing total fishing time it is assumed that the data obtained during census hours is representative of all hours included in the fishing day. The fishing day is defined broadly as the interval of the day during which fishing occurs. This is a variable quantity from day to day and changes with seasons. Some fishing is done at all hours of the day and night at some time during the season, but most anglers fish between seven o'clock in the morning and darkness. It is practical to conduct creel censuses during these hours of greatest activity, but rather

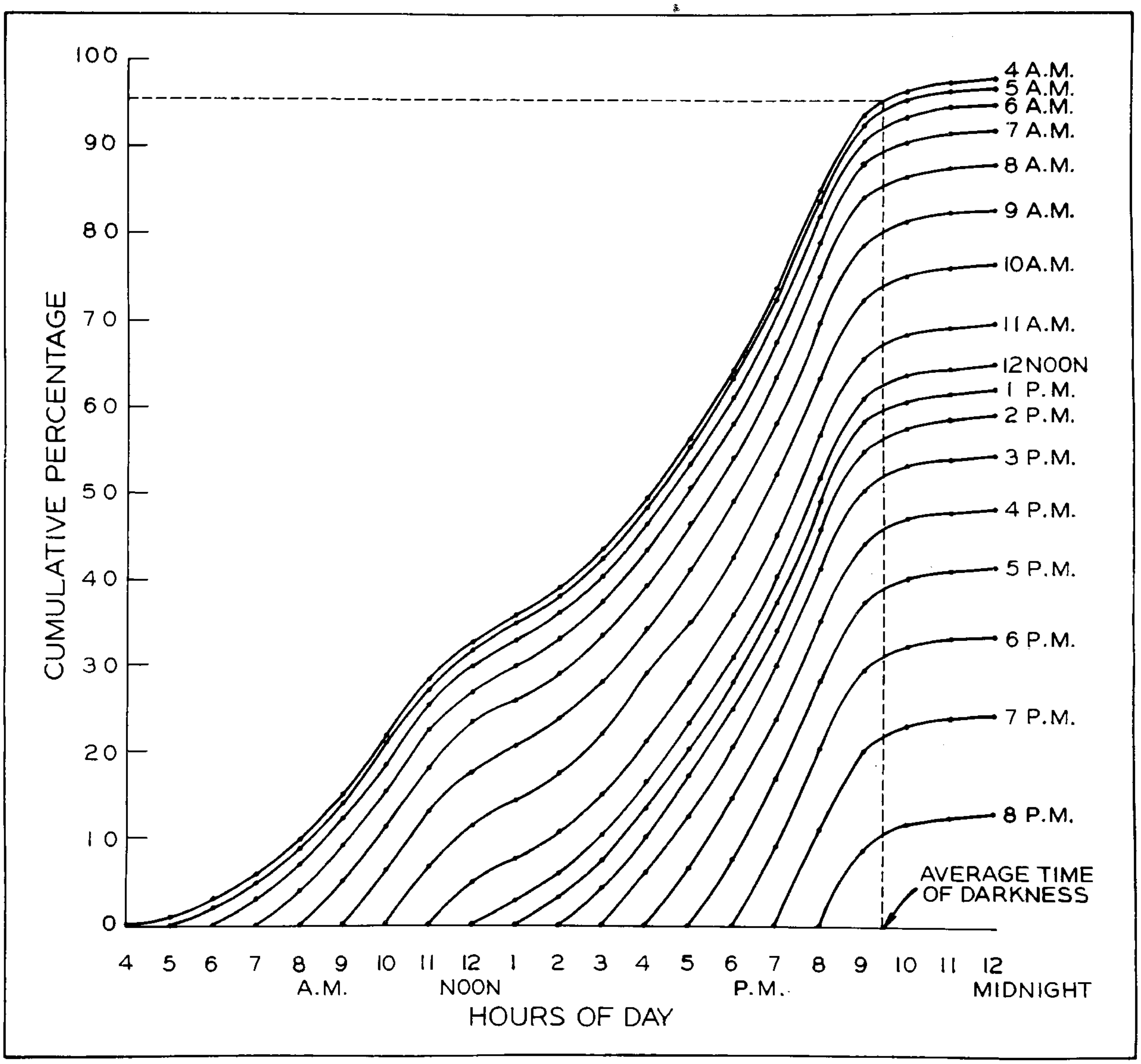
difficult to obtain records at night. Some adjustment may be necessary to compensate for the portion omitted from the census if complete yield data are required. Apparently most fishermen who arrive before the census clerk remain long enough to have an opportunity to appear in samples. Some error is introduced into the estimate of total angler hours by omitting the hours from darkness to seven in the morning. It is advantageous, however, to establish a constant value for length of day in hours for each season. Error may be introduced in individual calculations of total angler hours, but year-to-year data are more nearly comparable if constant time limits are used.

A basis for determining the best value for length of day is found in the percentage of angler hours omitted from various census intervals. If this portion is small, and if the catch rate during non-censused time is not different from the censused time, the estimate of total angler hours may be adjusted by the appropriate percentage omitted. This was determined graphically for the Bear Lake, 1941, data in the following way. Curves were constructed by plotting the cumulative percentage of all angler hours at various hours (Figure 12). Thus, the percentage of all angler hours included in a census starting at 8 a.m. and ending at time of darkness (9:30 p.m.) may be determined by referring to the curve labeled 8 a.m. and will be found to be 86 percent. This value is obtained by starting at the 9:30 p.m. point on the 8 a.m. curve and projecting across to the vertical scale on the left.

The length-of-fishing-day values used in the Intensive Creel Census are as follows.

Spring	14.0 hours
Summer	15.5 hours
Fall	13.5 hours
Winter	10.0 hours

Figure 12. Percentage of Total Angler Hours Included in Various Census Intervals, Bear Lake, 1941



CUMULATIVE PERCENTAGE
100
90
80
70
60
50
40
30
20
10
0
4 A.M.
5 A.M.
6 A.M.
7 A.M.
8 A.M.
9 A.M.
10A.M.
11 A.M.
12NOON
1 P.M.
2 P.M.
3 P.M.
4 P.M.
5 P.M.
6 P.M.
7 P.M.
8 P.M.
AVERAGE TIME OF DARKNESS
4 5 6 7 8 9 10 11 12 1 2 3 4 5 6 7 8 9 10 11 12
A.M.
NOON
P.M.
MIDNIGHT
HOURS OF DAY

Figure 12.

Thus, in compiling an estimate of total angler hours for the summer season the inflation is:

Boats per Hour	x	Anglers per Boat	x	15.5 Hours per Day	x	Number of Days in Season

For the Bear Lake data 15.5 hours (6 a.m. - 9:30 p.m.) includes 95 percent of all angler hours.

The Length of Fishing Season

The number of days included in each of the several seasons varies only slightly depending on the fishing regulations operating from year to year. The number of days included in each season for computational purposes in the Intensive Creel Census in recent years is approximately as follows.

Spring	56 days
Summer	83 days
Fall	76 days
Winter	77 days

The Discrepancy between Actual Total Angler Hours and Estimates of Total Angler Hours Based on Hourly Counts

It is a matter of practical convenience to make angler or boat counts on the clock-hour and at regular intervals. For an individual fishing trip the time recorded by counts may be considerably different from the true elapsed fishing time. If, for example, an angler fishes from 8:00 a.m. to 10:30 a.m. he would appear in the 8:00, 9:00, and 10:00 counts. If each time he is counted is considered equivalent to one angler hour his fishing time would appear to be 3 angler hours, but the actual elapsed time is

2 1/2 hours. Conversely, if he had fished from 8:15 a.m. to 9:45 a.m. he would appear in the 9:00 a.m. count only or one angler hour, but the elapsed time would be 1 1/2 hours. Boat counts are not made instantaneously. They may take as much as 1/2 hour to complete, depending on size of lake, etc. This tends to increase the probability that an individual angler arriving at or leaving the lake other than on the hour will correctly appear in an hourly count. It may thus be assumed with reasonable safety that the discrepancies in estimating the elapsed fishing time are compensating in the compilation of a seasonal estimate of the average number of anglers per hour.

Frequency of Boat Counts Required to Estimate Average Number of Boats per Hour

Several factors affect the number of boat counts that should be made on a lake to determine the average number of boats per hour. These include the variability of the numbers of boats present, the desired precision of estimating the average number, and the confidence level at which statements will be made about the precision attained, and also the practical considerations of the amount of time and money available for the census. The obvious objective of the Intensive Census is the most precise, unbiased estimate of the number of boats per hour possible within the limits of time and money available.

The number of hourly boat counts that are required to estimate the true average number of boats per hour for a given precision also determines the number of census days required in the boat count schedule. This obtains from the fact that only one count for a particular hour can be made each day. Boat counts could be made every 1/2 hour or 1/4 hour which would greatly increase the number of counts. However, this would not provide adequate sampling. The key point is that counts must be made on a considerable number of days to cover the variation from day to day, which will not be adequately covered by many counts on a few days. A study of sampling methods for estimation of the average number of boats per hour follows.

The complete census of Bear Lake in 1941 provides data for determination of the required number of times each hour should appear in a schedule of boat counts and also provides a basis for comparing methods of sampling.

Three methods for determining the number of counts required and for selecting these counts are:

(1) Strict random sampling in which each count of the 1162 possible counts has the same probability of being included in the sample.

(2) Stratified random sampling, whereby the total sample of N counts is divided equally among the 14 hours (8 a.m.-9 p.m.) during which boat counts are made, i.e., counts are made on N/14 days at 8 a.m. picked at random from all days of the season, and on N/14 days at 9 a.m. picked at random, etc.

(3) Stratified random sampling with the sample size of each hour in proportion to number of possible counts and variability of each hour stratum. This is an application of the method developed by Neyman and described by various authors (Snedecor, Cochran, and others).

Each of these methods was applied to the population of Bear Lake records for 1941 to learn which was most efficient. The precision attained, for several sample sizes, was the basis for comparison of the three methods. Actually, this study was reduced to a comparison of two methods of allocating sample size among the 14 hourly strata, because strict random sampling is not a very practical field operation. The results of random sampling, however, served as a basis of comparison for gain in precision of stratified sampling. As a corollary to this study by hours, a study of the efficiency of methods of sampling by days followed.

<u>Effect of Non-Normality (skewness) on Confidence Limits of Mean Number of Boats per Hour and Sample Size</u>. A sample consisting of N boat counts drawn at random from the population of all possible counts provides an estimate of the average number of boats per hour. Obviously, another sample of size N would not be expected to yield exactly the same mean. Repeated samples would form a distribution of mean values grouped about the true population mean. When confidence limits are calculated for the mean it is assumed that the means of repeated samples drawn from the population are normally distributed about the true mean. The distribution of the population of boat counts is obviously non-normal (Table 12), but this can be ignored if the sampling distribution of means approaches normality. It has been well established that such sampling distributions approach normality with increasing sample size. The conclusion of Carver (1930) is that if the sample N is fifty or larger and the population at least ten times N, the parent population can be ignored. The number of boat counts made in a season usually is greater than fifty, and the total number of possible boat counts exceeds ten times N so Carver's conditions seem to be satisfied.

Another "rule-of-thumb" for determining how large N is required for use of the normal approximation was suggested by Cochran (1953; p. 25) as:

$$N > 25G_1^2$$

where G_1 is a measure of skewness and is calculated as

$$\frac{1}{N\sigma^3} \sum_{i=1}^{N} (yi - \overline{Y})^3. \quad \text{(Cochran stated p. 25):}$$

"This rule is designed so that a 95 per cent confidence probability statement will be wrong not more than 6 per cent of the time."

$G_1 = 2.11$ was computed for the Bear Lake (1941) angler count data following the method of Cochran, and

$$N = 25\ G_1^2 = 112$$

Therefore, in order for the normal approximation to hold for the specified confidence limits of the mean, at least 112 counts should be included in samples drawn at random from the Bear Lake data. This requirement is easily met in drawing up boat count schedules.

Sample Size for Given Precision Assuming Random Sampling. Simple random sampling means that every possible hourly count during the season should have the same probability of being included in the sample consisting of N counts. Clearly, such random sampling is impractical when one considers the field application of these studies, but a first step in a comparison of sampling methods is a consideration of results of simple random sampling from this population. The question to be answered is--how many counts must be included in a sample to determine the average within prescribed limits if hours to be counted are picked entirely at random? The same question may then be asked about stratified random sampling.

Without previous experience to dictate needs, a precision is chosen in this study of 0.22. That is, the sample mean is within ±0.22 of the true mean unless a 1 in 20 chance has come off. If this precision is unnecessary or unattainable it can be relaxed later. The equation

$$N = \frac{t^2 S^2}{(\bar{X} - M)^2}$$ (Snedecor, 1948: p. 457)

gives the required sample size.

The average and variances of all counts (see Table 12) are

$\bar{X}$ = 4.47 anglers/hour

S^2 = 21.87

and the sample size at the 5 percent level is

$$N = \frac{(1.96)^2(21.87)}{(0.22)^2} = 1735 \text{ counts}$$

whereas, a precision of ±0.45 of the mean requires only

$$N = \frac{(1.96)^2(21.87)}{(0.45)^2} = 420$$

The values 0.22 and 0.45 for precision of estimating the means are equal to 5 and 10 percent of the sample mean. This does not mean that N determined here will be adequate for 5 percent or 10 percent of all mean values.

Apparently, a precision of ±5 percent of this mean is not attainable assuming random sampling of these data for there are only 1162 counts possible. A precision of ±10 percent of this mean, suggesting a required random sample size of N = 420, will therefore be a starting point and any reduction in the sample size or improvement in precision possible by stratification will be considered a gain in efficiency.

<u>Correction for Finite Population.</u> Instead of computing the variance of the mean of a random sample from the relationship $S^2 = \frac{\sigma^2}{N}$ which applies to an infinite population, Cochran (1953: p. 17) suggested the added factor of $(N-n)/N$ for the variance to correct for a finite population. He stated also in the same place, "In practice the fpc [fpc is the finite population correction] can be ignored whenever the sampling fraction does not exceed 5 per cent, and for many purposes even if it is as high as 10 per cent. The effect of ignoring the correction is to overestimate the standard error of the estimate $\bar{y}$." In the present study, the sampling ratio is considerable, and thus the appropriate formulae for standard error of the mean and for confidence limits are:

Standard Error of Mean: $$Sy = \frac{S}{\sqrt{n}} \sqrt{\frac{N - n}{N}}$$ (Cochran, 1953: p. 19)

Confidence Limits: $$\bar{Y} = \bar{y} \pm \frac{ts}{\sqrt{n}} \sqrt{\frac{N - n}{N}}$$ (Cochran, 1953: p. 20)

The correction factor $\frac{N-n}{N}$ also affects the technique for estimating random sample size. By the method suggested by Cochran (p. 56) a first approximation (N_o) to the required sample is made in the usual manner.

$$N_o = \frac{t^2 S^2}{(\bar{X} - M)^2} = 420$$ (in this study) If the sampling ratio is negligible then $N_o = 420$ counts is the appropriate sample size assuming random selection of hours on which to make the counts.

The sampling ratio $\frac{420}{1162}$ is large, however, and the correction for finite population is needed.

$$N = \frac{N_o}{1 + \frac{N_o}{N}} \quad \text{(Cochran, 1953: p. 56)}$$

$$N = \frac{420}{1 + \frac{420}{1162}} = 309$$

Thus, the apparent required sample size, assuming random sampling, is reduced appreciably by applying Cochran's finite population correction.

Gain in Efficiency by Stratified Random Sampling. From a theoretical approach a gain in efficiency (increase in confidence limits of estimate of means for a given sample size) may result from stratified sampling if a heterogeneous population can be separated into strata which are more homogeneous within themselves than the total population.

The hours of the day are convenient intervals for boat counts and form natural strata for analysis of variation. Furthermore, stratified sampling is more applicable to the field operation of a creel census than strict random sampling.

This possibility of a gain in efficiency was explored in the Bear Lake data by tallying angler counts from the original complete census by hours of the day, and computing the mean, variance, and standard deviation for each hour. The distribution of angler counts by hours, the averages for hours and respective variances, and certain other data from Bear Lake, 1941 are included in Table 12. For the purposes of this study the behavior of statistics of angler counts was assumed to be the same as for boat counts. Thus, the numbers of anglers on the lake at 8 a.m. on 83 different days form the distribution of counts in the 8 a.m. stratum. Each of these hourly strata will contribute to the total variance. If the variances of the individual strata differ greatly, each stratum should be sampled separately to reduce the total variance. The resulting total variance of the mean, or more properly, the standard error of the mean, is a basis for deciding whether or not such treatment is desired. The equation for the variance of the mean estimate for stratified random sampling is

$$\text{Var. } \bar{Y} = V(\bar{y}_{st}) = \frac{1}{N^2} \sum_{h=1}^{1} N_h \,(N_h - n_h)\, \frac{S_h^{\,2}}{n_h} \quad \text{(Cochran, 1953: p. 69)}$$

For example, values for the Bear Lake counts are:

$$\text{Var. } \bar{y} = \frac{1}{(1162)^2} \sum 83\ (83 - 22)\ \frac{S_h^{\,2}}{22} = 0.0453$$

where $S_h^{\,2}$ is the variance of each hourly column of Table 12. This computation was greatly simplified because the N_h were all equal to 83 days, and by the further assumption of equal allocation of the total sample size $N = 309$ among the 14 hours. The 95 percent confidence interval for the mean for stratified random sampling is

$\bar{y} - 1.96\sqrt{0.0453} < \bar{Y} < \bar{y} + 1.96\sqrt{0.0453}$ and the confidence limits are $\bar{y} \pm 0.42$. This result indicates that stratified random sampling from Bear Lake 1941 data produced improved confidence limits over strict random sampling with the same total sample size (Table 13).

Table 12. Frequency Distribution of Anglers by Hours, Bear Lake, 1941

Number Anglers	8 AM	9 AM	10 AM	11 AM	12 N	1 PM	2 PM	3 PM	4 PM	5 PM	6 PM	7 PM	8 PM	9 PM	Total Freq.
0	25	22	19	17	25	37	27	18	8	11	5	3	...	9	226
1	11	8	9	7	9	11	13	12	12	6	6	5	2	8	119
2	11	15	9	13	15	14	13	15	17	13	10	11	8	8	172
3	7	7	11	8	7	2	9	7	9	10	11	9	3	9	108
4	10	4	7	6	8	5	9	11	7	9	10	6	8	3	103
5	6	9	2	6	5	6	5	4	7	4	9	5	7	5	80
6	3	2	3	4	3	3	...	6	3	6	8	8	6	8	63
7	4	3	6	5	3	1	3	4	5	6	3	7	11	5	66
8	1	4	4	4	1	...	1	...	6	5	4	4	2	4	40
9	1	1	4	3	...	...	1	1	2	1	4	7	8	2	35
10	1	1	...	...	1	1	...	2	...	2	4	3	4	2	21
11	1	2	2	1	...	...	1	...	1	1	2	4	6	3	24
12	...	...	...	1	1	1	...	...	1	2	...	2	4	4	16
13	...	1	2	3	1	1	1	2	1	1	...	1	1	5	20
14	1	1	1	1	...	...	...	...	1	2	2	1	3	...	13
15	1	...	...	1	...	...	...	1	1	1	2	1	3	1	12
16	...	1	...	1	2	...	...	...	...	2	2	2	1	2	13
17	...	...	1	...	...	...	...	...	...	...	1	1	2	2	7
18	...	...	...	...	...	1	...	...	1	1	...	...	1	...	4
19	...	1	1	...	...	...	...	...	...	...	1	1	1	1	6
20	...	...	1	...	...	...	...	...	...	...	...	...	2	1	4
21	...	...	...	1	1	...	...	...	...	...	...	1	...	1	4
22	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
23	...	1	...	...	...	...	...	...	1	...	...	...	...	...	2
24	...	...	...	...	...	...	...	...	...	....	...	...	...	...	...
25	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
26	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
27	...	...	...	...	...	..1	...	...	...	...	...	1	...	...	2
28	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
29	...	...	1	1	...	...	...	...	...	...	...	...	...	...	..2
30	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
Total	82	83	83	83	83	83	83	83	83	83	83	83	83	83	1,162
$\overline{X}$	2.86	3.75	4.49	4.63	3.36	2.08	2.28	3.14	4.35	4.89	5.61	6.64	8.14	6.36	4.472
s^2	10.1	20.0	27.5	26.7	22.4	10.2	6.8	10.2	17.9	18.7	18.9	25.8	22.3	28.8	21.87
s	3.18	4.47	5.24	5.17	4.73	3.19	2.61	3.19	4.23	4.32	4.35	5.08	4.72	5.37	4.68

Table 13. Comparison of Standard Errors and 95 Per Cent Confidence Limits of Mean Number of Anglers Per Hour by Three Methods of Sampling for Several Sample Sizes, Bear Lake, 1941 Data

Total Sample Size N	Strict Random			Stratified Random with Equal Allocation			Stratified Random with Allocation Proportional to N and σ		
	$s_{\bar{x}}^2$	$s_{\bar{x}}$	Conf. Limits	$s_{\bar{x}}^2$	$s_{\bar{x}}$	Conf. Limits	$s_{\bar{x}}^2$	$s_{\bar{x}}$	Conf. Limits
70	0.312	0.559	±1.10	0.2550	0.505	±0.99	0.2417	0.492	±0.96
140	0.121	0.348	±0.68	0.1195	0.346	±0.68	0.1140	0.337	±0.66
308	0.052	0.228	±0.45	0.0453	0.213	±0.42	0.0428	0.207	±0.41
420	0.033	0.182	±0.36	0.0289	0.170	±0.33	0.0273	0.165	±0.32
700	0.012	0.111	±0.22	0.0108	0.104	±0.20	0.0097	0.098	±0.19

Allocation of Sample Size Among Hourly Strata. There are theoretical grounds for allocating the total sample size among the various hour strata in proportion to the variability of the respective strata. Kenney and Keeping (1951: p. 155) attribute the development of these methods to J. Neyman as follows: "It has been shown by J. Neyman[16] that in stratified sampling the variance of the mean is least (and therefore the mean is most accurately estimated) if, for a fixed total sample size, the n_i are proportional to $N_i \sigma_i$. If this condition is satisfied, and if the cost of taking the sample is proportional to the sample size, the sampling is optimum, in the sense that the greatest possible accuracy is obtained for a fixed cost." Thus, the sample size in a stratum should be proportional to the size and to the standard deviation of the stratum. It is somewhat intuitive that a larger sample is needed in a more variable population than in a less variable one.

This foregoing theory may have important application in determining the required number of boat counts if there are considerable differences in variability of counts between hours or between days. The gain in efficiency may warrant estimating the variances of the strata separately and allocating the total sample size accordingly.

The merits of this method were partially determined by allocating sample sizes in the Bear Lake data in proportion to the respective values of σ. (Table 14). The procedure followed here is more fully developed by Snedecor (p. 460) and Cochran (p. 74).

The equation for variance of the mean of stratified random sampling already used in the preceding section is applicable here also. The N_h values are constant in this case because the strata are all the same size because 83 counts were possible at each hour of the day for the season as a whole. Another minor difference is that the calculation involves separate N_h entries for each stratum (Table 14). The value for variance of the mean obtained by the equation for a total sample of $n = 309$ is

$$\text{Var}(\bar{y}_{st}) = \frac{1}{(1162)^2} \sum 83\,(83 - n_h)\,\frac{s_h^2}{n_h} = 0.0428$$

which allows confidence limits of

$$\bar{Y} \pm 1.96\sqrt{0.0428} \text{ or } \bar{y} \pm 0.41,$$

which in turn is a relatively small increase in efficiency over stratified random sampling with equal size strata.

Small gains in efficiency were noted for a total sample of 309 in favor of stratified sampling with allocation by hours. Other total sample sizes were treated in the same way to see if gains were stable over different amounts of census effort (Tables 13 and 14). The total sample N = 140 approximates the number of boat counts usually made on the Intensive Creel Census in recent years.

<u>Gains in Efficiency by Stratification of Counts by Days</u>. The range of variances of angler counts by days as calculated for the Bear Lake 1941 data (Table 15) suggests that greater gains in sampling efficiency are possible through stratification by days and by allocation of the census effort than were possible when the stratification was by hours. Accordingly, the total sample of 308, which is the approximate number of counts necessary for a precision of $\bar{X} \pm 0.45$ assuming strict random sampling, was allocated among the various days equally and in proportion

Table 14. Allocation of Total Sample Among Hours in Proportion to Standard Deviation, Bear Lake, 1941

Hour	Number of Possible Counts N_h	Standard Deviation σ	$N_h\sigma$	Size of Sample $n_h = N_h\sigma(n/S)$				
8 A. M.	83	3.18	263.95	4	7	17	22	37
9 A. M.	83	4.47	371.01	5	11	23	31	52
10 A. M.	83	5.24	434.92	6	13	27	37	61
11 A. M.	83	5.17	429.11	6	12	27	36	60
12 Noon	83	4.73	392.59	6	11	24	33	55
1 P. M.	83	3.19	264.77	4	7	16	22	38
2 P. M.	83	2.61	216.63	3	6	14	18	31
3 P. M.	83	3.19	264.77	4	7	16	22	38
4 P. M.	83	4.23	351.09	5	10	22	30	49
5 P. M.	83	4.32	358.56	5	10	22	31	51
6 P. M.	83	4.35	361.05	5	10	23	31	51
7 P. M.	83	5.08	421.64	6	12	26	36	59
8 P. M.	83	4.72	391.76	6	11	24	33	55
9 P. M.	83	5.37	445.71	6	13	28	38	63
Total	1,162		S=4,968	n=70	140	309	420	700

Table 15. Frequency Distribution of Anglers by Days, Bear Lake, 1941

Number of Anglers	Sunday (1)	Monday (2)	Tuesday (3)	Wednesday (4)	Thursday (5)	Friday (6)	Saturday (7)	Holidays	Total Freq.
0	10	50	32	40	40	36	15	3	226
1	12	20	17	16	19	23	11	1	119
2	10	34	38	31	20	19	19	1	172
3	11	6	14	18	21	21	14	3	108
4	22	13	15	9	10	14	18	2	103
5	12	7	8	14	14	8	16	1	80
6	10	6	11	9	4	4	15	4	63
7	20	7	7	3	6	6	14	3	66
8	4	5	7	3	5	5	7	4	40
9	7	1	2	4	7	3	10	1	35
10	4	1	2	2	6	1	4	1	21
11	7	1	...	1	5	3	4	3	24
12	3	...	1	1	3	2	5	1	16
13	8	...	...	2	...	...	4	6	20
14	5	1	...	...	1	3	2	1	13
15	4	1	...	...	4	...	2	1	12
16	4	1	...	...	3	...	2	3	13
17	1	...	...	1	...	1	2	2	7
18	3	...	...	...	...	1	...	...	4
19	3	...	...	...	...	2	1	...	6
20	1	...	...	...	...	2	1	...	4
21	3	...	...	...	...	...	...	1	4
22	...	...	...	...	...	...	...	...	...
23	1	...	...	...	...	...	1	...	2
24	...	...	...	...	...	...	...	...	...
25	...	...	...	...	...	...	...	...	...
26	...	...	...	...	...	...	...	...	...
27	..1	...	...	...	...	...	1	...	2
28	...	...	...	...	...	...	...	...	...
29	2	...	...	...	...	...	...	...	2
30	...	...	...	...	...	...	...	...	...
Total	168	154	154	154	168	154	168	42	1,162
$\bar{X}$	7.607	2.552	2.961	2.981	4.012	3.734	5.869	8.95	
s^2	36.82	9.40	7.00	9.78	17.38	19.64	12.79	28.14	
s	6.06	3.07	2.65	3.13	4.16	4.43	3.58	5.30	

to the standard deviation (Table 16). Several other total sample sizes were given the same treatment to provide future reference material (Table 17). The variance of the mean and confidence limits were then calculated. Significant improvements in the confidence limits apparently are possible if the season is stratified by days and the total number of counts is allocated in proportion to the size and the standard deviation of the strata. This gain in efficiency can be expressed as narrower confidence limits of ±0.37 or an equivalent reduction in the total number of counts required if the confidence limits remain at ±0.45. The variance of the mean for stratified sampling provides the necessary information for the new sample size.

$$s^2 = Ns_{\bar{x}}^2 = 308\ (0.036) = 11.1$$

and

$$N = \frac{t^2 s^2}{(\bar{x} - m)^2} = \frac{(1.96)^2(11.1)}{(0.45)^2} = 210.4$$

Thus, 210 counts by stratified random sampling with equal allocation would produce the same precision of estimating the mean number of anglers per hour as 308 counts by strict random sampling. Similarly, 239 counts would be necessary if the method of stratified sampling with equal allocation among days was used. The number of days required to complete the counts will depend on the number of counts made each day (see Table 18).

<u>The Precision Actually Attained in Estimating the Average Number of Boats per Hour in the Intensive Creel Census.</u> Experimental results just presented indicate themost efficient sampling method, the procedures

Table 16. Standard Error of Mean and Confidence Limits of Mean Number of Anglers Per Hour. Total Sample Stratified by Days With Equal Allocation and With Allocation in Proportion to N and σ, Bear Lake, 1941

	Strict Random Sampling	Number of Hourly Counts in Season	
		Equal Allocation	N σ - Allocation
Sunday	...	42	69
Monday	...	42	32
Tuesday	...	42	28
Wednesday	...	42	32
Thursday	...	42	47
Friday	...	42	46
Saturday	...	42	40
Holiday	...	14	14
Total n	308	308	308
$s_{\bar{x}}^2$	0.052	0.041	0.036
$s_{\bar{x}}$	0.228	0.202	0.190
Confidence Limits	±0.450	±0.400	±0.370

Table 17. Allocation of Total Sample Size (Number of Counts) Among Days in Proportion to Standard Deviation, Bear Lake, 1941

Day	Number of Possible Counts N_h	Standard Deviation σ	$N_h \sigma$	Size of Sample $n_h = N_h \sigma (n/S)$				
Sunday	168	6.06	1,018.1	15	30	69	93	150
Monday	154	3.07	472.8	8	16	32	43	80
Tuesday	154	2.65	408.1	6	12	28	37	60
Wednesday	154	3.13	482.0	7	14	32	44	70
Thursday	168	4.16	698.9	11	22	47	64	110
Friday	154	4.43	682.2	10	20	46	63	100
Saturday	168	3.58	601.4	9	18	40	55	90
Holiday	42	5.30	222.6	4	8	15	21	40
Total	1,162		S=4,586.1	n=70	140	309	420	700

Table 18. Number of Days Necessary to Complete Required Counts for Several Levels of Census Effort, Bear Lake, 1941

		Required Number of Counts	Number of Days		
			14 Counts Per Day *	7 Counts Per Day	4 Counts Per Day
Sunday	(1)	69	5	10	18*
Monday	(2)	32	3	5	8
Tuesday	(3)	28	2	4	7
Wednesday	(4)	32	3	5	8
Thursday	(5)	47	4	7	12
Friday	(6)	46	4	7	12
Saturday	(7)	40	3	6	10
Holiday	(8)	14	1	2	4

* Only 12 Sundays Available in Season

for predicting the number of boat counts required, and the proper method of analyzing data. There remains the question of the area and degree of adjustment necessary in future creel censuses in light of these findings. Information about the precision actually attained by the schedules now in use must first be available.

The boat count information from the Intensive Creel Census for Pontiac Lake for the summer seasons of 1947 through 1952 is presented in the Appendix. These counts were actually stratified by days of the week and so I computed the mean, and variances of days as strata. These data provided the values necessary for computing the variance of the mean of stratified sampling and confidence limits of the mean by Cochran's formulas (Table 19). Applications of the results are subject to limitations, however. The variances (S^2) of the strata measure only the internal variation of the sample, and must be considered as uncertain estimates of the true population variance (σ^2). Several sources of bias are apparent in the sampling method which reflect doubt upon the assumption that S^2 is an unbiased estimate of σ^2. First the selection of hours to make boat counts is not a random process (the selection of succeeding counts on the same day results in lower total variance estimates than by random selection); and second, very little is known about the behavior of variance estimates based on small samples drawn from non-normal parent populations. The importance of these facts is that although exact confidence limits can be constructed from the sample variances, the probability associated with those limits may deviate considerably from the assumed 95 percent level.

The sample variances obtained in various years on Pontiac Lake show a fairly consistant relationship among days of the week (Table 20).

Table 19. Mean Number of Boats Counted Per Hour, Standard Error of the Mean and Confidence Limits as Computed from Intensive Creel Census Data for Pontiac Lake in the Years 1947 through 1952

Year	Number of Counts	Mean Number Boats Per Hour	Variance of Mean ($s_{\bar{x}}^2$)	Standard Error of Mean ($s_{\bar{x}}$)	Confidence Limits of Mean
1947	100	53.7	4.58	2.140	±4.2
1948	84	51.8	7.72	2.778	±5.4
1949	63	37.3	6.99	2.644	±5.2
1950	51	32.2	2.39	1.546	±3.0
1951	38	37.3	6.48	2.545	±5.0
1952	73	18.0	6.53	2.554	±5.0

Table 20. Sample Variances of Boat Counts on Pontiac Lake, 1947 to 1952

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1947	2069	66	93	164	192	90	1519
1948	998	399	173	247	176	1565	1266
1949	1342	172	398	144	80	65	1171
1950	62	66	917	94	0.1	9	37
1951	647	260	39	375	10	12	493
1952	1156	54	22	4	198	1265	1012
Sum	6274	1017	1642	1028	656	3006	5498
Average Variance	1046	170	274	171	109	501	916

The average variance of the estimates from the six-years, 1947 to 1952, provides estimates of the strata variances for future prediction purposes and for allocation of samples.

Problems of Estimating the Total Catch of Fish

The total number and weight of fish removed from a lake are measures of the fishing success of anglers when considered with the amount of fishing effort expended. Furthermore, knowledge of the amount of fish removed in relation to the population of fish present in a body of water is essential to the management of many fisheries. For example, a study of the total catch of predator fish, such as the pike and bass, may indicate that more of these fish are being removed than is desirable even though the catch per unit of fishing effort is very low. It is usually more convenient to obtain information about the number of fish caught by anglers than about the weight of fish although weight knowledge is valuable.

In the Intensive Creel Census the total weight of fish removed is estimated through periodic samples of fish weights, and estimates of the total catch in numbers of the various species of fish. Creel census clerks weigh and measure the length of fish in the anglers' catch when it is convenient to do so. The problems of sampling to determine the average weights of fish are beyond the scope of this paper, but the problems of deriving estimates of the total catch in numbers are considered.

Method of Deriving an Estimate of Total Number of Fish Caught from Estimates of the Average Catch per Hour and Total Angler Hours

The basic procedure for estimating total catch is a simple inflation of the average catch per hour over all hours of fishing in the season.

$$\text{Average Catch per Hour per Angler} \times \text{Total Number of Angler Hours} = \text{Total Catch}$$

Each of the factors in this inflation is an estimate obtained from samples. It is necessary to review how each of these factors is obtained. The average catch per hour per angler is the arithmetic average of all catch per hour values computed for individual anglers who were contacted in the sampling program. For illustration, several such catch-per-hour values with confidence limits are presented here for Pontiac Lake (Table 21).

An estimate of the total number of angler hours involves boat counts and the average number of anglers per boat during the season. The inflation is as follows:

$$\text{Average Number Anglers per Boat} \times \text{Average Number of Boats per Hour} \times \text{Length of Fishing Day in Hours} \times \text{Number of Days in Season} = \text{Total Angler Hours}$$

The number of hours in the season (15 hours per day x 83 days) is a definite figure and is relatively constant from year to year, but the average number of anglers per boat and the average number of boats per hour are variables that must be estimated from samples. The problems of sampling to determine the average number of boats per hour have been discussed in preceding sections.

<u>Sampling to Determine the Number of Anglers per Boat</u>

The number of anglers sharing a boat while fishing varies from one to five or more, but on the average this quantity is quite uniform over the various hours of the day and from day to day, and is not correlated with total fishing intensity (Figure 13). Thus, the average number of anglers per boat may be determined quite precisely with little concern about bias or inaccuracies from disproportionate sampling. Angler contact records provide the necessary data for estimating this quantity. In the Intensive Census procedure the individual catches of all of the anglers in one boat are recorded on one creel census slip. Thus, on each slip an observation of the number of anglers per boat is available. The average number of anglers per boat is the arithmetic average of all such observations. As long as angler contacts are accomplished in a schedule which represents all hours of the day and each day of the week this average will be an unbiased estimate of the true mean number of anglers per boat. This is true because there are no discernible trends of increase or decrease in the number of anglers per boat with time. What is needed, however, is a measure of the variability of this quantity to allow confidence statements about the mean and estimates of the required sample size. The variance of the individual observations of anglers per boat is an unbiased estimate of the true variance, and therefore can be used to compute the standard error of the mean. Standard errors and mean values of the number of anglers per boat were computed for the summer season from the angler contact records for several years on Pontiac Lake (Table 22). These values indicate the precision attained in estimating this quantity, and provide data for inflation to estimates of the total catch of fish.

Confidence Limits for Estimate of Total Angler Hours

The variance of the product of anglers per boat times boats per hour is arrived at by a somewhat different process than used in attaining the variances of the individual factors. The method of

Table 21. Summary of Average Catch per Hour per Angler for Summer Seasons During Several Years on Pontiac Lake

Year	Number of Contacts	Σx	Σx^2	Average Catch per Hour per Angler	Sample Variance s^2	Variance of the Mean $s_{\bar{x}}^2$
1947	1465	2235	7162	1.53	2.56	0.00175
1948	967	973	2571	1.01	1.64	0.00170
1949	471	405	1020	0.86	1.43	0.0030
1950	335	277	620	0.83	1.17	0.0035
1951	344	307	689	0.89	1.21	0.0035
1952	471	561	2003	1.19	2.84	0.0060

Figure 13. Relationship between Number of Anglers and Number of Boats, Pontiac Lake, 1947 to 1952

Figure 14. Relationship between Catch per Hour per Angler and Number of Boats per Hour, Pontiac Lake, 1947 to 1952

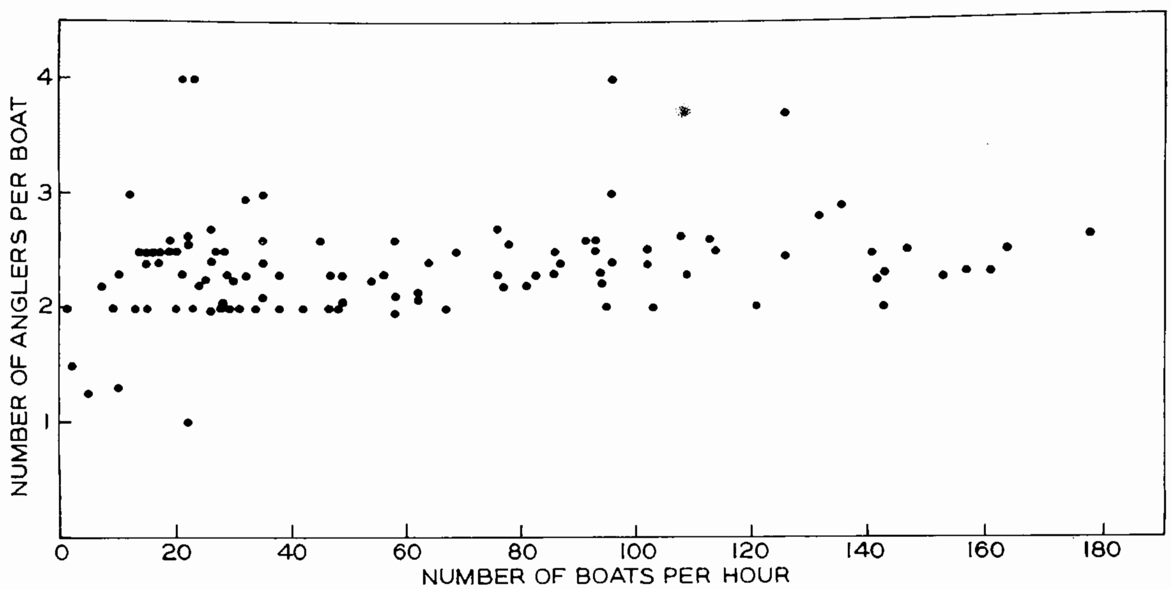


Figure 13.

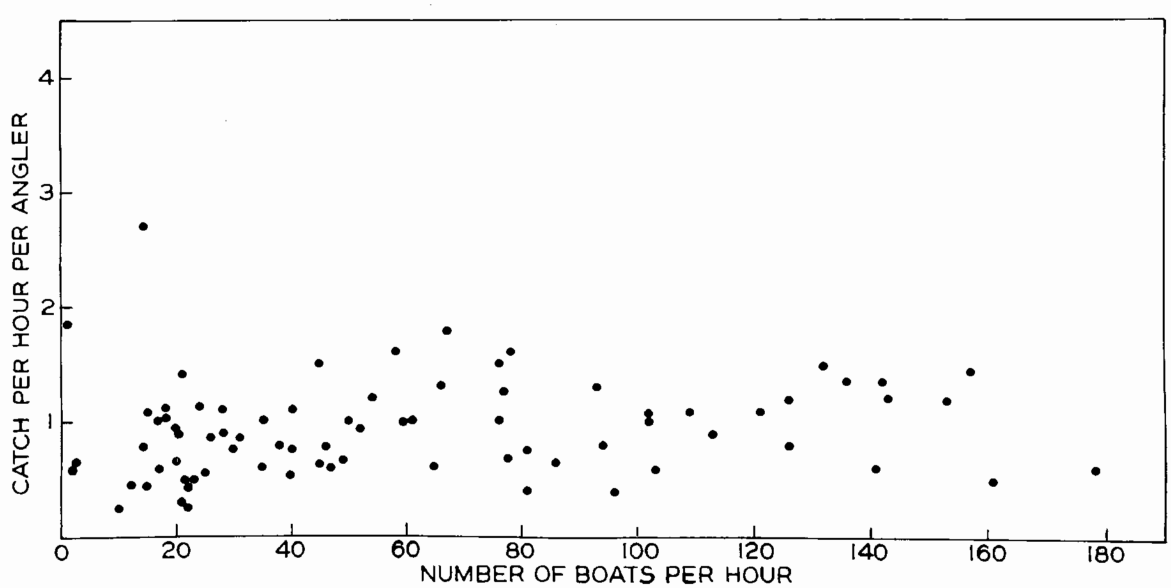


Figure 14.

Table 22. Average Number of Anglers per Boat During the Summer Season on Pontiac Lake for Several Years as Determined from Angler Contacts

Year	Number Boats in Contacts N	Number Anglers in Contacts Σx	Σx^2	Average Number Anglers per Boat $\bar{x}$	s^2	$s_{\bar{x}}^2$	Standard Error of Mean $s\bar{x}$	Confidence Limits $1.96s_{\bar{x}}$
1947	624	1463	3903	2.30	0.76	0.0121	0.110	0.22
1948	415	968	2564	2.30	0.74	0.0177	0.133	0.26
1949	202	471	1271	2.30	0.86	0.0424	0.206	0.40
1950	150	334	855	2.23	0.65	0.0433	0.208	0.41
1951	147	339	1309	2.31	3.60	0.0243	0.156	0.31
1952	207	424	978	2.05	0.53	0.0256	0.160	0.31

computing the variance of the product of two factors when each is independently subject to sampling error was reviewed by Schumacher and Chapman (1942: p. 193). They showed that if $\bar{M}$ and $\bar{N}$ are independently distributed, and $V(\bar{M})$ and $V(\bar{N})$ are estimates of their sampling variances, then

$$V(\bar{M}\bar{N}) = \bar{M}^2 \left[V(\bar{N})\right] + \bar{N}^2 \left[V(\bar{M})\right] + \left[V(\bar{M})\right] \left[V(\bar{N})\right]$$

in which $V(\bar{M}\bar{N})$ is an estimate of the sampling variance of $\bar{M}\bar{N}$. The third term in this expression may be ignored in the application at hand because the product $\left[V(\bar{M})\right] \left[V(\bar{N})\right]$ is trivial when compared with the contribution of the first two terms to the variance $V(\bar{M}\bar{N})$.

The assumption that the sampling errors of the means $\bar{M}$ and $\bar{N}$ are independent and approximately normally distributed does not seem unreasonable for this study. It is possible that under certain conditions the number of anglers per boat could be related to the number of boats available on a lake. If this occurred the number of anglers per boat would not be independent of the number of boats per hour and the above approximation to the variance of a product would be a hazardous procedure. Similarly, if the average catch per hour per angler were a function of the intensity of boats the assumption of independence would be invalid. However, the data available for Pontiac Lake do not indicate any relationship between either the number of anglers per boat and boats per hour or between catch per hour per angler and boats per hour (Figures 13 and 14). The values plotted in these figures are the average number of anglers per boat and the average catch per hour per angler with the corresponding number of boats observed. Although the hypothesis of independence is not refuted in this example, it should be re-examined whenever dependence is suspected in census data, possibly for each separate analysis of total catch. For a more theoretical discussion the reader is referred to Craig (1936), Aroian (1947), and Calhoun (1950). Calhoun gives an example of an application of the method to estimating the total catch of fish.

In the present problem, the average number of boats per hour and the average number of anglers per boat are equivalent to N and M respectively, and the above equation can be expressed as follows.

$$\text{Variance of Product of Mean Number of Anglers per Boat} \cdot \text{Mean Number of Boats per Hour} = \left[\text{Mean Anglers per Boat}\right]^2 \left[\text{Variance of Mean Boats per Hour}\right] + \left[\text{Mean Boats per Hour}\right]^2 \left[\text{Variance of Mean Anglers per Boat}\right]$$

The product of anglers per boat times boats per hour multiplied by the total number of hours of fishing in the season yields an estimate of the total number of angler hours spent on the lake (Table 23) This total ranges from about 50,000 to 160,000 angler hours for the summer seasons on Pontiac Lake (1947 to 1952).

<u>Confidence Limits for Estimate of Total Catch</u>

The product of total angler hours times average catch per hour now becomes an estimate of total number of fish caught in the season. However, for ease in computing products and variances of products the total catch may be derived in three steps: (1) calculate the average and variance of anglers per hour from the product (anglers per boat) times (boats per hour), and (2) multiply anglers per hour by the average catch per hour to get the total catch per hour, and finally, (3) multiply the total catch per hour by the total number of hours in the season. The following example may serve to clarify the procedures.

Table 23. Estimates of the Total Number of Angler Hours Spent on Pontiac Lake During the Summer Seasons of 1947 to 1952

Year	Estimate of Total Angler Hours	Standard Error	Confidence Limits 95%
1947	158,800	4,500	154,300-163,300
1948	153,200	12,100	141,100-165,300
1949	110,000	12,600	97,400-122,600
1950	92,300	9,600	82,700-101,900
1951	110,900	10,600	100,300-121,500
1952	47,500	7,700	39,800-55,200

The information needed to derive an estimate of total catch of all fish from Pontiac Lake during the summer season in 1947 (Table 24), is available in Tables 23, 25, and 26.

(1) Mean Number of Anglers per Boat x Mean Number of Boats per Hour = Angler Hours per Hour

$2.3 \times 53.7 = 123.5$ angler hours per hour

Variance of the product $= (2.3)^2(4.58) + (53.7)^2(0.0121) = 59.2$

(2) Angler Hours per Hour x Mean Catch per Hour per Angler = Catch per Hour

$123.5 \times 1.53 = 188.9$ Fish per Hour

Variance of the product $= (123.5)^2(0.00175) + (1.53)^2(59.2) = 153.9$

Standard error of product $= \sqrt{153.9} = 12.4$

(3) Total Number of Hours in Season = 15.5 Hours per Day x 83 Days = 1286

Total Catch = 1286 hours x 188.9 fish per hour = 243,000 fish

1286 x Standard Error of Product = 15,946

Confidence Limits $(\pm 2S_{\bar{x}})$ for Total Catch = 243,000 ± 2 x 16,000 approximately.

Table 24. Estimates of Total Catch of Fish During the Summer Season for Several Years on Pontiac Lake

Year	Total Catch of Fish	Standard Error	Confidence Limits
1947	243,000	15,946	211,000-275,000
1948	154,320	13,760	140,560-168,080
1949	94,894	12,410	82,484-107,304
1950	76,633	9,709	66,924-86,342
1951	98,636	11,510	87,126-110,146
1952	56,455	9,876	46,579-66,331

SUMMARY AND CONCLUSIONS

In this paper the methods used in Michigan for collecting and analyzing catch data from lakes have been studied with the objective of determining efficient sampling procedures. There are several compelling reasons for developing efficient and accurate methods for conducting creel censuses. Creel studies are an important source of facts relating to fish and the conservation of fisheries. A considerable portion of fisheries research funds are currently devoted to creel census projects. Much time, effort, and money is expended in measuring the yield of fish to anglers. Usually, creel censuses are part of a continuing study of experimental lakes and streams which are thought to be representative of most waters in the state. Long range studies are necessary to evaluate the effects of management attempts. The results of experiments on these test lakes influence management policies and procedures on many lakes. This wide application of results and the need for continuous data require that the best possible information be obtained in the creel studies. Of course, accurate results could be achieved on the test waters through a complete census of all fishing activity, and in some situations this will be necessary. However, the need for catch information has expanded greatly. New problems about sport fisheries are posed every year, and very often a census is called for. In order to meet the increasing demand on available funds and personnel it is important that themost efficient sampling methods suitable for intensive censuses be determined.

Several conclusions and recommendations are suggested from this present analysis for the conduct of the Intensive Creel Census and the subsequent analysis of catch records.

Probably the most important suggestion to be made here is for adoption of stratified random sampling with optimum allocation of the total sample among days. Stratified sampling techniques will be useful in two ways:

(1) To more accurately measure the precision actually attained by present census methods (the present methods of analyzing records do not recognize stratified sampling actually in use).

(2) To obtain the best precision possible for a given amount of census effort.

Actually, the framework of the present census schedule is essentially that of stratified sampling by days. All that is needed is the application of appropriate formulae and the proper allocation of census effort to take advantage of the gains in efficiency possible by this method. Present procedures for scheduling angler interviews and boat counts should be modified to include the following:

(a) Some decision must be made about the precision of estimating desired before a schedule is set up for a census. Confidence limits (95%) of ±0.1 fish per hour have been selected arbitrarily as the desired precision of estimating the average catch per hour. This seems to be a useful figure, although for some lakes this precision will not be attainable during seasons of low fishing intensity. This is true because the variability of catch-per-hour data may require that more records be obtained in the sample than there are anglers present during the season.

(b) An advance estimate of the variance of catch-per-hour-per-angler records must be available for the lake in question. This can come from earlier censuses of the same lake, or from a preliminary sample. The number of angler contacts required and the precision attained in estimating the mean catch are functions of the variance of the mean.

(c) From this estimate of the population variance an estimate of the total sample required is possible. The formula $N = \frac{t^2 s^2}{(\bar{x} - m)^2}$ is appropriate (see also Figure 5 of this thesis). If the sampling ratio is not negligible, i.e., if the sample is large in proportion to the total number of anglers, a correction for finite population is needed. This will reduce the apparent required sample size.

(d) The total sample size should be allocated among the various days in proportion to the fishing intensity and variability of catch-per-hour-per-angler records on the different types of days (strata). The variance of the mean will be the least if the population of anglers is stratified by days, and the total sample allocated among the strata. Reduction of the variance of the mean catch by stratification is equivalent to either an increase in precision of estimating or a reduction in the total number of contacts required. Information about the numbers of anglers likely to fish on the lake and estimates of the strata variances are necessary. This data can come only from previous experience with similar lakes or from the preliminary samples. It should be remembered that the predictions of sample size are based on estimates of the variance, and some margin of safty should be allowed for error. The distribution of variances drawn from a non-normal parent population such as catch per hour per angler is not well known and errors can occur when the distribution is extremely skew.

(e) The precision actually attained in the census should be calculated using formulas for variance of the mean and confidence limits of the mean for stratified sampling (see page 28 of this thesis), so these quantities will be available for subsequent predictions of required sample size, etc.

Probably the most rewarding area for an increase in the precision attained in the Intensive Census is in the scheduling of boat counts. Studies of the required sample size, and the precision attained in estimating boats per hour for Bear and Pontiac lakes indicate that more counts are desirable. Some improvement will be possible if the total number of boat counts needed to attain the desired precision is allocated among days according to the variability of counts and intensity of fishing on the various days. An improvement in the precision of estimating boats per hour is important because this quantity is a factor in the inflation to estimates of total fishing intensity and the total catch of fish.

Application of these findings ought not to cause major changes in methodology in the Intensive Creel Census. The most obvious need is for increased sampling effort on Sundays and other days of high angler intensity and possibly less effort on other days.

The methods of compiling and analyzing records in the laboratory should also be modified to include provision for weighting catch-per-hour records for fishing intensity. Arithmetic errors in the computation of the average catch per hour may result if the records are not properly weighted. These errors are considerable when fishing intensities are high such as are encountered at Pontiac Lake.

The records studied did not reveal any bias in the method of sampling to determine the average number of anglers per boat. Ordinarily the number of angler contacts necessary to determine the average catch per hour will be more than adequate to determine the number of anglers per boat. It will be necessary, however, to tabulate the individual records of anglers per boat to obtain information about the sampling error associated with this quantity.

Statistical methods for assigning confidence limits have been applied to the various estimates of the Intensive Creel Census. In doing this, certain assumptions are made regarding randomness of sampling and the distribution and independence of variates. These assumptions are reasonable for census material but should be reviewed for each major study.

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Appendix A. Daily and Seasonal Summations of Catch per Hour per Angler, Number of Anglers, and Ratios of Boats Contacted to Boats Counted for Various Hours on Pontiac Lake 1948 to 1951

Appendix A

Table 1. Daily and Seasonal Summations of Catch per Hour per Angler, Number of Anglers, and Ratios of Boats Contacted to Boats Counted for Various Hours on Pontiac Lake in 1948

	8 A. M.	9 A. M.	10 A. M.	11 A. M.
Daily Summations of Catch per Hour per Angler (and Number of Anglers Contacted)	7.56 (10)	4.31 (10)	16.57 (19)	4.31 (6)
	1.86 (1)	2.99 (13)	1.86 (3)	2.99 (13)
	68.82 (58)	4.32 (11)	38.42 (41)	35.99 (44)
	0.92 (4)	25.46 (35)	2.93 (10)	7.61 (17)
	4.00 (9)	38.50 (60)	4.00 (8)	23.09 (34)
	... ...	... ...	... ...	... ...
	... ...	... ...	... ...	... ...
	... ...	... ...	... ...	... ...
	... ...	... ...	... ...	... ...
	... ...	... ...	... ...	... ...
	... ...	... ...	... ...	... ...
Seasonal Total	83.16	75.58	63.78	73.99
Total Anglers	(82)	(129)	(81)	(114)
Average Catch per Hour per Angler	1.014	0.586	0.787	0.649
Ratio of Number of Boats Contacted to Number of Boats Counted by Days	4/14	5/21	7/26	3/20
	1/1	5/10	2/2	5/22
	25/143	24/81	4/20	20/94
	2/9	5/7	2/21	7/15
	3/12	15/86	18/161	15/77
	...	...	...	...
	...	...	...	...
	...	...	...	...
	...	...	...	...
	...	...	...	...
	35/17	...	...	...
Total Contacted	35	54	33	50
Total Counted	179	205	230	228

Table 1 (continued)

12 Noon	1 P. M.	2 P. M.	3 P. M.	4 P. M.
66.12 (61)	19.82 (17)	68.10 (41)	24.56 (25)	75.94 (36)
12.91 (15)	17.75 (23)	12.91 (13)	8.85 (11)	0.14 (8)
2.96 (6)	15.87 (25)	2.36 (21)	17.21 (24)	55.81 (45)
19.28 (23)	10.29 (15)	13.06 (12)	0 (2)	2.14 (5)
16.93 (15)	5.34 (12)	12.00 (8)	5.14 (7)	6.60 (8)
56.01 (50)	11.17 (17)	3.00 (3)	17.81 (18)	38.77 (38)
4.00 (8)	0.93 (6)	13.79 (18)	0 (1)	... ...
... ...	9.63 (24)	72.62 (53)	3.69 (6)	... ...
... ...	... ...	2.14 (8)	... ...	... ...
... ...	....	10.82 (11)	... ...	... ...
... ...	... ...	41.21 (39)	... ...	... ...
177.61	90.80	252.01	77.26	179.40
(178)	(122)	(208)	(69)	(140)
0.998	0.744	1.212	1.120	1.281
24/102	8/28	16/78	12/38	15/64
6/28	10/38	5/35	4/32	4/31
4/10	11/49	2/9	11/35	19/153
8/126	5/35	5/95	1/34	2/17
6/19	6/23	3/17	3/29	4/28
24/121	0/65	1/32	0/54	16/102
2/23	7/17	8/30	8/25	...
...	3/13	23/142	1/22	...
...	10/96	3/19	3/103	...
...	...	6/26	...	...
...	...	17/109	...	...
74	60	89	43	60
429	364	592	372	395

Table 1 (concluded)

5 P. M.	6 P. M.	7 P. M.	8 P. M.	9 P. M.
41.83 (36)	42.91 (27)	40.33 (34)	40.56 (29)	32.71 (33)
13.80 (10)	17.14 (20)	5.80 (7)	41.90 (40)	4.68 (8)
2.00 (3)	51.18 (36)	4.68 (14)	27.02 (20)	... ...
17.37 (16)	8.95 (16)	30.92 (21)	6.81 (12)	... ...
... ...	9.90 (13)	... ...	3.30 (7)	... ...
... ...	77.45 (49)	... ...	65.98 (39)	... ...
... ...	... ...	... ...	... ...	... ...
... ...	... ...	... ...	... ...	... ...
... ...	... ...	... ...	... ...	... ...
... ...	... ...	... ...	... ...	... ...
... ...	... ...	... ...	... ...	... ...
75.00	207.53	81.73	185.57	37.39
(65)	(161)	(76)	(147)	(41)
1.154	1.289	1.075	1.262	0.912
16/54	10/76	16/58	10/136	15/61
4/27	9/49	3/61	17/76	4/42
2/5	15/164	6/47	8/93	...
7/24	6/21	8/45	5/25	...
...	17/132	...	4/21	...
...	...	...	16/114	...
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...
...	...	...	...	...
29	57	33	60	19
110	442	211	465	103

Appendix A

Table 2. Daily and Seasonal Summations of Catch per Hour per Angler, Number of Anglers, and Ratios of Boats Contacted to Boats Counted for Various Hours on Pontiac Lake in 1949

	8 A. M.	9 A. M.	10 A. M.	11 A. M.
Daily Summations of Catch per Hour per Angler (and Number of Anglers Contacted)	52.22 (41) 4.87 (12)	22.31 (17) ... (2) 10.83 (10) 13.05 (20) 19.58 (14) 12.76 (18)	49.72 (36) 4.87 (12) ... (4)	4.38 (10) 5.82 (6) ... (4) 2.49 (8) 9.50 (14) ... ...
Seasonal Total	57.09	78.53	54.59	22.19
Total Anglers	(53)	(81)	(52)	(42)
Average Catch per Hour per Angler	1.077	0.969	1.040	0.528
Ratio of Number of Boats Contacted to Number of Boats Counted by Days	17/69 5/17	7/126 1/1 5/15 8/16 6/21 9/47	15/58 5/33 1/2	4/147 3/17 2/27 3/19 7/67 ...
Total Contacted	22	36	21	19
Total Counted	86	226	93	277

Table 2 (continued)

12 Noon	1 P. M.	2 P. M.	3 P. M.	4 P. M.
43.80 (20)	3.20 (2)	9.07 (7)	7.84 (24)	42.29 (44)
0 (7)	1.66 (2)	29.23 (36)	1.10 (6)	18.55 (19)
0 (4)	0 (2)	9.63 (16)	5.75 (13)	27.67 (31)
... ...	0.99 (6)	7.32 (21)	0 (4)	5.39 (16)
... ...	3.75 (12)	0 (3)	... ...	... ...
... ...	3.00 (2)	8.15 (20)	... ...	... ...
43.80	12.60	63.40	14.69	93.90
(31)	(26)	(103)	(47)	(110)
1.413	0.485	0.616	0.313	0.854
11/58	1/143	3/51	11/19	15/54
3/37	1/1	12/63	2/26	8/16
1/1	1/44	7/12	7/7	15/28
...	2/16	10/31	2/5	7/94
...	6/29	1/39	...	...
...	1/48	10/66	...	...
15	12	43	22	45
96	281	262	57	192

Table 2 (concluded)

5 P. M.	6 P. M.	7 P. M.	8 P. M.
6.53 (15)	47.64 (40)	12.49 (14)	32.60 (35)
3.10 (11)	23.95 (24)	1.77 (10)	22.85 (25)
... ...	27.58 (29)	0 (1)	3.11 (8)
... ...	3.23 (12)	... ...	... ...
... ...	... ...	... ...	... ...
... ...	....	... ...	... ...
9.63	102.40	14.26	58.56
(26)	(105)	(25)	(68)
0.370	0.975	0.570	0.861
8/27	15/108	6/69	13/113
6/39	8/39	5/41	11/41
...	13/34	1/5	3/93
...	5/87	...	...
...	...	...	...
...	...	...	...
14	41	12	27
66	268	119	247

Appendix A

Table 3. Daily and Seasonal Summations of Catch per Hour per Angler, Number of Anglers, and Ratios of Boats Contacted to Boats Counted for Various Hours on Pontiac Lake in 1950

	8 A. M.	9 A. M.	10 A. M.	11 A. M.
Daily Summations of Catch per Hour per Angler (and Number of Anglers Contacted)	8.66 (35) 4.52 (7) 5.86 (7)	57.94 (40) 8.12 (5) 3.62 (6)	10.30 (29) 23.93 (23) 4.86 (9)	41.93 (24) 5.54 (7) ... ...
Seasonal Total	19.04	69.68	39.09	47.47
Total Anglers	(49)	(51)	(61)	(31)
Average Catch per Hour per Angler	0.389	1.366	0.641	1.531
Ratio of Number of Boats Contacted to Number of Boats Counted by Days	15/83 3/15 4/11	17/157 3/7 3/3	12/92 10/18 4/14	10/221 3/2 ... ...
Total Contacted	22	23	26	13
Total Counted	109	167	124	223

Table 3 (continued)

12 Noon	1 P. M.	2 P. M.	3 P. M.	4 P. M.
2.00 (2) 23.93 (26) 2.20 (5)	7.54 (9) 25.65 (14) 2.18 (4)	20.17 (28) 2.56 (9) 4.50 (2) 6.72 (18) 7.07 (10)	18.00 (13) 0.90 (6) 22.40 (37)	10.00 (19) 1.18 (4) 3.46 (13) 5.07 (7) ... ...
28.13	35.37	41.02	41.30	19.71
(33)	(27)	(67)	(56)	(43)
0.852	1.310	0.612	0.737	0.458
1/78 11/19 2/18	3/276 7/17 2/5	12/31 4/18 1/17 9/6 7/8	6/21 2/6 14/41	10/28 2/27 7/16 5/11 ...
14	12	33	22	24
115	298	80	68	82

Table 3 (concluded)

5 P. M.	6 P. M.	7 P. M.	8 P. M.
7.98 (7) 0.90 (6) 22.82 (32)	22.86 (32) 0 (2) 1.42 (4)	4.32 (3) 0.90 (6) 14.74 (27)	27.32 (27) ... ...
31.60	24.28	19.96	27.32
(45)	(38)	(36)	(27)
0.702	0.639	0.554	1.012
4/19 2/5 12/37	15/21 1/19 2/9	2/16 2/6 10/31	14/20 ... ...
18	18	14	14
61	49	53	20

Appendix A

Table 4. Daily and Seasonal Summations of Catch per Hour per Angler, Number of Anglers, and Ratios of Boats Contacted to Boats Counted for Various Hours on Pontiac Lake in 1951

	8 A. M.	9 A. M.	10 A. M.	11 A. M.
Daily Summations of Catch per Hour per Angler (and Number of Anglers Contacted)	20.21 (33) 5.86 (12) 15.55 (14)	62.91 (40) 57.52 (21) 8.48 (8)	13.72 (24) 5.86 (11) 16.32 (24)	63.98 (38) 48.52 (28) 3.81 (5) ... ...
Seasonal Total	41.62	128.91	35.90	116.31
Total Anglers	(59)	(69)	(59)	(71)
Average Catch per Hour per Angler	0.705	1.868	0.608	1.638
Ratio of Number of Boats Contacted to Number of Boats Counted by Days	13/141 6/19 6/17	16/58 9/14 4/8	9/178 5/21 12/18	15/67 12/14 3/16 ... ...
Total Contacted	25	29	26	30
Total Counted	177	80	217	97

Table 4 (continued)

12 Noon	1 P. M.	2 P. M.	3 P. M.	4 P. M.
6.54 (9)	26.43 (17)	19.26 (21)	3.97 (9)	... ...
13.13 (21)	0.67 (1)	33.02 (21)	... ...	1.28 (2)
... ...	... ...	21.27 (21)	... ...	15.54 (19)
... ...	... ...	4.34 (4)	... ...	... ...
... ...	... ...	3.06 (7)	... ...	... ...
19.67	27.10	80.95	3.97	22.28
(30)	(18)	(74)	(9)	(26)
0.696	1.506	1.094	0.441	0.857
4/11	6/55	9/56	4/33	3/56
11/21	1/11	10/31	...	1/57
...	...	10/19	...	9/13
...	...	2/4	...	...
...	...	4/25	...	...
15	7	35	4	13
32	66	135	33	126

Table 4 (concluded)

5 P. M.		6 P. M.		7 P. M.		8 P. M.		9 P. M.	
16.98	(20)	11.50	(20)	44.09	(24)	1.60	(5)	...	...
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...
...	...	...	...	...	...	...	...	...	...
16.98		11.50		44.09		1.60		...	...
	(20)		(20)		(24)		(5)		...
0.849		0.575		1.837		0.12		...	
8/37		8/17		9/48		2/12		...	
...		...		...		...		...	
...		...		...		...		...	
...		...		...		...		...	
...		...		...		...		...	
8		8		9		2		...	
37		17		48		12		...	

Appendix B. Method of Computing Skewness Factor (α_3)

APPENDIX B

Method of Computing Skewness Factor (α_3)

The following method for computing the skewness factor of the distribution of a variate (X) is taken from a lecture by Professor H. C. Carver at the University of Michigan. There are three steps to follow in the computation:

Step 1. Accumulate summations of X, X^2 and X^3. A calculator and a table of cubes greatly facilitates this operation.

Step 2. The equation for skewness (α_3) is

$$\alpha_3 = \frac{N^2 \cdot \sum X^3 - 3N \cdot \sum X \cdot \sum X^2 + 2(\sum X)^3}{\left[N \cdot \sum X^2 - (\sum X)^2\right] \sqrt{N \cdot \sum X^2 - (\sum X)^2}}$$

which can be rewritten as

$$\alpha_3 = \frac{C}{A\sqrt{A}}$$

where

$$A = N \cdot \sum X^2 - (\sum X)^2$$

$$B = N \cdot \sum X^3 - \sum X \sum X^2$$

$$C = N \cdot B - 2A \cdot \sum X$$

Step 2. consists of first computing A and B and then inserting the values obtained for A and B in the equation for C.

Step 3. Compute $\alpha_3 = \frac{C}{A\sqrt{A}}$ = skewness factor.

Appendix C. Stratification of Boat Counts by Days. Intensive Creel Census of Pontiac Lake, 1947 to 1952

Appendix C

Table 1. Stratification of Boat Counts by Days. Intensive Creel Census of Pontiac Lake, 1947

	Number of Boats Counted						
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	161	4	14	21	39	31	116
	189	8	22	29	37	26	133
	149	7	30	32	41	17	170
	146	16	36	21	54	24	123
	155	6	7	26	18	0	134
	144	13	13	21	19	25	163
	180	21	16	34	16	22	156
	146	23	26	38	14	6	78
	101	18	1	53	17	11	132
	118	16	17	10	22	20	101
	128	21	23	11	...	19	126
	98	18	13	18	...	27	128
	89	33	26	15	...	32	94
	71	...	...	7	...	...	64
	50	...	...	7	...	...	67
	61	...	...	...	...	...	58
	57	...	...	...	...	...	...
	51	...	...	...	...	...	49
n_h	18	13	13	15	10	13	18
Σx^2	278,782	3,994	5,690	10,141	9,397	6,282	235,999
Σx	2,094	204	244	343	277	260	1,945
$\bar{x}$	116.3	15.7	18.8	22.9	27.7	20.0	108.1
s^2	2,069	66	93	164	192	90	1,519

Appendix C

Table 2. Stratification of Boat Counts by Days. Intensive Creel Census of Pontiac Lake, 1948

	Number of Boats Counted						
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	143	30	14	28	21	102	121
	161	31	26	38	20	78	109
	126	49	28	54	35	64	102
	95	76	35	58	34	76	132
	142	12	49	61	1	136	114
	153	21	35	10	2	9	81
	164	23	5	22	10	20	94
	93	32	47	23	9	19	65
	86	...	42	29	...	17	54
	77	...	19	26	...	17	38
	96	...	17	28	...	25	32
	103	...	21	24	...	24	27
	...	...	25	21	...	45	61
	...	...	7	...	...	...	...
	...	...	15	...	...	...	...
	...	...	13	...	...	...	...
	...	...	22	...	...	...	...
n_h	12	8	17	13	8	13	13
Σx^2	183,539	12,176	13,148	16,660	3,408	49,502	96,802
Σx	1,439	274	420	422	132	632	1,030
$\bar{x}$	119.9	34.3	24.7	32.5	16.5	48.6	79.2
s^2	998	399	173	247	176	1,565	1,266
Number counts possible (N_h)	168	168	168	168	154	168	168

Appendix C

Table 3. Stratification of Boat Counts by Days. Intensive Creel Census of Pontiac Lake, 1949

	Number of Boats Counted						
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	126	31	69	12	1	1	63
	147	28	58	16	1	4	54
	143	34	58	39	11	3	108
	66	41	51	17	26	8	113
	94	21	19	33	21	15	16
	87	19	27	37	3	17	27
	93	16	69	39	2	22	44
	47	5	...	...	2	...	29
	67	4	...	...	2	...	7
	48	5	...	...	1	...	39
	...	...	...	...	2	...	41
n_h	10	10	7	7	11	7	11
Σx^2	96,346	5,706	19,941	6,189	1,266	1,088	38,331
Σx	918	204	351	193	72	70	541
$\bar{x}$	91.8	20.4	50.1	27.6	6.5	10	49.2
s^2	1,342	172	398	144	80	65	1,171
Number counts possible (N_h)	168	168	154	154	154	168	168

Appendix C

Table 4. Stratification of Boat Counts by Days. Intensive Creel Census of Pontiac Lake, 1950

	Number of Boats Counted						
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	83	17	157	31	6	7	23
	92	21	221	28	5	13	19
	78	19	276	21	6	10	27
	72	16	6	0	...	...	18
	41	3	2	15	...	...	11
	37	2	14	18	...	...	14
	31	5	12	19	...	...	18
	...	...	6	17	...	...	29
	...	...	16	8	...	...	...
	...	...	19	11	...	...	...
	...	...	12	9	...	...	...
	...	...	...	0	...	...	...
n_h	7	7	11	12	3	3	8
Σx^2	30,632	1,385	150,843	3,651	97	318	3,425
Σx	434	83	741	177	17	30	159
$\bar{x}$	62.0	11.9	67.4	14.8	5.7	10	19.9
s^2	62	66	917	94	0.1	9	37
Number counts possible (N_h)	154	154	154	154	140	140	140

Appendix C

Table 5. Stratification of Boat Counts by Days. Intensive Creel Census of Pontiac Lake, 1951

	Number of Boats Counted						
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	141	33	58	56	19	17	31
	178	37	67	56	13	18	57
	151	48	55	2	17	21	49
	120	8	...	0	12	25	14
	...	16	...	19	...	...	14
	...	11	...	21	...	...	0
	...	...	...	11	...	...	...
	...	...	...	4	...	...	...
	...	...	...	5	...	...	...
	...	...	...	6	...	...	...
	...	...	...	8	...	...	...
n_h	4	6	3	11	4	4	6
Σx^2	88,766	5,203	10,878	7,340	963	1,679	7,003
Σx	590	153	180	188	61	81	165
$\bar{x}$	147.5	25.5	60	17.1	15.3	20.3	27.5
s^2	647	260	39	375	10	12	493
Number counts possible (N_h)	140	154	154	154	140	140	140

Appendix C

Table 6. Stratification of Boat Counts by Days. Intensive Creel Census of Pontiac Lake, 1952

	Number of Boats Counted						
	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	69	14	19	2	18	78	9
	83	19	18	3	21	84	4
	88	21	18	2	40	63	8
	29	11	12	6	37	51	11
	1	5	15	2	21	2	74
	4	13	21	1	27	3	69
	9	0	11	5	18	1	52
	4	0	5	...	0	2	9
	7	5	14	...	0	1	4
	12	6	10	...	0	1	5
	10	...	...	...	3	3	...
	...	...	...	...	1	0	...
	...	...	...	...	0	...	...
n_h	11	10	10	7	13	12	10
Σx^2	20,642	1,374	2,261	83	5,238	19,739	16,505
$\bar{x}$	28.7	9.6	14.3	3.0	14.3	24.1	26.5
Σx	316	96	143	21	186	289	265
s^2	1,156	53.6	21.6	4.3	198	1,265	1,012
Number counts possible (N_h)	154	154	140	140	140	140	154